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**Rayleigh Wave Assessment of Damage  
and Integrity of Mine Structures**

By Michael J. Friedel and Richard E. Thill

UNITED STATES DEPARTMENT OF THE INTERIOR



BUREAU OF MINES

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### UNIT OF MEASURE ABBREVIATIONS USED IN THIS REPORT

cm	centimeter	kHz	kilohertz
g/cm <sup>3</sup>	gram per cubic centimeter	km/s	kilometer per second
Hz	hertz	m	meter
J/cm <sup>2</sup>	joule per square centimeter	min	minute
kg/m <sup>3</sup>	kilogram per cubic meter	pct	percent

# RAYLEIGH WAVE ASSESSMENT OF DAMAGE AND INTEGRITY OF MINE STRUCTURES

By Michael J. Friedel<sup>1</sup> and Richard E. Thill<sup>2</sup>

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## ABSTRACT

The integrity of mine structures, such as the roofs, ribs, face and supporting pillars, is difficult to assess beyond the exposed surface. To mitigate potential rockfall hazards, the U.S. Bureau of Mines is assessing the usefulness of Rayleigh wave dispersion analysis to detect damage and stress-relieved zones in mine structures. Classical surface wave dispersion modeling is utilized at both laboratory and field scales in damaged and undamaged material to assess the integrity of a mine pillar and roof. To more fully exploit Rayleigh wave propagation characteristics, derivation of a novel dispersion parameter was conducted using a convolutional model. The development of a suitable Rayleigh wave dispersion system based on insight derived from forward modeling is presented. The application of inverse modeling for damage and stress assessment in the mine environment is also discussed.

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## INTRODUCTION

Loose rock causes instability in mine structures and endangers miners with rockfalls, failure of support structures, and falls of ground from highwalls. Rock masses, in which mines are developed, often are discontinuous and contain systems of joints, fractures, and faults. These combined with in situ stress conditions and the disturbance of stress fields caused by the mine openings create hazardous conditions for potential falls of ground or catastrophic structural failure of mine roof, rib, or floor. These ground control problems not only have serious consequences regarding the safety of miners, but also can adversely affect the economics of the mining operation, considering such costs as compensation for injuries and death, lost time, production loss, lost or damaged equipment, and extra support and cleanup costs. Sometimes portions of the mine, or the entire mine has to be shutdown because of "bad ground" conditions. Obviously, for safety and economic reasons, it is in the best interest of mining operations to detect bad ground conditions in advance of mining and take appropriate measures to avoid ground falls and failures of mine support structures. Unfortunately, fractures, flaws or damage created by blasting, excavation processes, and stress relief in mine structures may go undetected by visual examination by even the most experienced miners. Often the fractures and flaws in mine structures are not visibly evident and are hidden at depth. New methods and technology are required to detect and delineate fractures and zones of damage in mine structures for the prevention of rockfalls and control of structural instability hazards.

To mitigate potential hazards such as rockfalls, caving, spalling, and structural failure, the U.S. Bureau of Mines is assessing the use of Rayleigh wave dispersion analysis for detecting fragmentation damage (overbreak) and stress relief fractures in-mine structures. This approach applies to both classical and novel surface wave dispersion techniques to determine the presence and pervasiveness of fractures in rock, thereby permitting the assessment of structural integrity of the damaged rock. Geophysical technology now used by the geotechnical and mining industry relies mainly on the use of seismic body (compressional and shear) waves to assess characteristics of the rock mass (1-4),<sup>3</sup> since elastic wave velocities are functions of the elastic properties and density of the rock. Typically, these methods seek to interpret changes in wave traveltime or velocity in terms of the rock mass properties. While the seismic refraction and reflection surface methods are currently the most established methods for this purpose in

mining, these have limitations for applications in the assessment of the integrity of mine structures. For example, the seismic refraction technique relies on the measurement of traveltimes of head waves and on inverse modeling to determine the depth to refractor, thickness, and elastic wave velocity of subsurface layers. The refraction technique fails, however, when a higher speed strata overlies lower speed rock units; i.e., when a velocity inversion is encountered. This is a common occurrence in the mining environment (3-4). The seismic reflection method is designed to operate from the Earth's surface at frequencies that are generally insensitive to small-scale features such as fractures in the mine structures. To operate underground on mine structures, entirely new seismic concepts and instrumentation would need to be developed. Other seismic technology developed in recent times includes crosshole seismic profiling and three-dimensional seismic profiling (5-6). These techniques are difficult to interpret, however, when refracted head waves overtake body waves, and also require expensive, high-energy sources, and borehole drilling for access.

The surface wave propagation technique, on the other hand, doesn't require borehole access and, moreover, can operate with a much lower power source, since the bulk of energy from an impact pulse is partitioned into surface wave energy. Surface wave instrumentation should be amenable to operating over the limited space occupied by typical supporting mine structures. Although surface wave technology appears to provide a welcome alternative for operating in-mine structures for detecting hidden fractures and damage, its feasibility must be thoroughly evaluated both in computer simulations and in field experiments for specific applications in the mine environment. Surface wave dispersion first must be modeled for the anticipated depth of penetration, range of frequencies, sensitivity, and propagation distances required for evaluating the small- to medium-scale fractures encountered in mine structures.

This report presents the results of mathematical modeling and feasibility analyses, using input from laboratory and field experimental results, as the first phase in the evaluation of Rayleigh wave dispersion technology for flaw detection in-mine structures. The basic approach utilizes application of classical and novel surface wave dispersion techniques to assess the structural integrity of damaged and undamaged rock at laboratory and field scales. Input parameters for the models are derived from data obtained at both laboratory and field scales. Follow-on efforts will develop and field test Rayleigh wave generation-detection instrumentation that can suitably operate in the mine environment.

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<sup>3</sup>Italic numbers in parentheses refer to items in the list of references at the end of this report.



## RAYLEIGH WAVE DISPERSION

Impacting the surface of a layered elastic medium results in the propagation of a variety of seismic waves including head (direct and refracted compressional and shear), body (reflected compressional and shear), and surface wave (Rayleigh, Love, channel and Stoneley) modes. The greatest portion of transmitted seismic energy (about 70 pct), however, is partitioned into the Rayleigh wave mode (7). Rayleigh waves arise as a direct consequence of constructive interference between compressional and shear waves at the surface of a body. The nature of Rayleigh wave particle motion is retrograde elliptical and in a vertical plane. This plane polarized phenomenon is characterized by both vertical and longitudinal vector components (8).

Surface waves are dispersive in the presence of velocity anisotropy. Specifically, there exists a variation of particle velocity with frequency, or wavelength in a layered media. The relationship between frequency, wavelength, and velocity is given by

$$\lambda = \frac{V_r}{f} \quad (1)$$

where  $\lambda$  = Rayleigh wavelength, m,

$V_r$  = Rayleigh velocity, km/s,

and  $f$  = frequency, Hz, where  $2\pi f$  = radians per second.

At high frequencies, surface wave displacements penetrate only to shallow depths; whereas, lower frequencies penetrate to intermediate and deeper layers. While there is no theoretical limit to the penetration depth of any wavelength, from a practical standpoint, the energy is confined within approximately one wavelength from the displaced surface (9). Hence, Rayleigh wave propagation at a particular frequency is expected to have a particle velocity characteristic of the rock at a penetration depth of one

wavelength. Moreover, the dependency of surface wave velocity on frequency (or wavelength) results in a unique and characteristic dispersion relationship for the medium traversed. In the case when velocity increases with depth, the low frequency Rayleigh wave energy arrives in advance of the higher frequency energy, within the dispersed Rayleigh wave packet. This phenomenon is referred to as normal dispersion. Conversely, when velocities decrease with depth, the lower frequency energy will arrive later in the Rayleigh wavetrain. In media where the velocity structure is mixed, as in damaged or stressed mine structures, dispersion of the Rayleigh wave energy will occur and frequencies will be mixed.

The Rayleigh pulse represents a complete duration of the dispersed waveform. Within the pulse, the corresponding distance per unit time traveled by a point of constant phase (individual peaks or troughs) is the phase velocity,  $V_p$ . The conventional relationship is given by

$$V_p(\omega) = \frac{\omega}{1/\lambda} \quad (2)$$

where  $\omega$  = angular frequency =  $2\pi f$ .

The velocity at which the packet of frequencies travels is known as the group velocity,  $V_g$  and is expressed as

$$V_g(\omega) = \frac{\delta\omega}{\delta k}, \quad (3)$$

where  $k$  = wavenumber =  $\omega/V$ .

The group velocity is dependent on the rate at which the phase velocity changes with frequency. Generally, these classical Rayleigh wave parameters, i.e., phase velocity and group velocity, will not have the same velocity unless the medium is homogeneous and isotropic. This ideal condition is not likely to occur in most mining environments.

## ASSESSMENT BY CLASSICAL RAYLEIGH WAVE DISPERSION

Initially, classical techniques, developed earlier by earthquake seismologists for the assessment of Rayleigh wave dispersion in large-scale Earth structures, were applied to the problem of investigating geologic features in-mine structures at various scales. In particular, the problem of assessing damaged mine structures using Rayleigh waves was investigated. Site-specific forward dispersion methods to assist in model development were used to predict the capabilities of Rayleigh waves for detecting the depth

of damage or stress relief in mine structures. The site-specific rock properties for the Rayleigh wave dispersion model, moreover, were considered essential for the optimum design of both the source and recording system. These help determine the range of frequencies and relative amplitude gain for various scales of investigation that are crucial for the development of a prototype system that can operate in the mining environment.

## FORWARD MODELING

Haskell (10) presented a solution in the 1950's for the forward modeling problem of determining phase and/or group velocities associated with a particular structure. The relevant boundary value problem satisfies the Hookean-elastic equations of motion within the media, subject to the vanishing of all displacements at depth, continuity of displacements and stress across interfaces, and the vanishing of traction on the free surface. In the Thomson-Haskell technique, the allowed phase velocities occur at the sign changes or zeros in a characteristic function (11). The characteristic function is dependent on the model parameters, frequency band selected, and trial phase velocity.

The computer program Rayleigh wave dispersion (RWD) (12) used for these studies provides a solution for

the forward model, consisting of a finite number of horizontal layers overlying a semi-infinite half-space. Individual layers are composed of materials that are considered homogeneous and isotropic. User defined material parameters include layer thickness, density, and body wave velocity. While the RWD program can handle velocity inversions, it becomes inoperable when a shear velocity of zero magnitude is used, for example, as with a water-filled void. In addition to the material property input, the user must specify a beginning frequency, frequency increment, total number of roots to calculate, trial phase velocity (greater than the actual value), and convergence error. The resulting output comprises a sequence of both Rayleigh wave (phase) and group velocity functions.

## THEORETICAL DISPERSION ALONG GRANITE BLOCK SURFACE

### INTACT ROCK MODEL

The physical model used to evaluate dispersion characteristics resulting from Rayleigh wave propagation along an undamaged surface of Barre granite appears in figure 1. This model demonstrates the lack of effect that a homogeneous, isotropic model has on Rayleigh wave dispersion. In this model, a compressional velocity of 4.29 km/s, shear velocity of 2.81 km/s, and density of 2.64 g/cm<sup>3</sup> were used, based on data obtained from an earlier report (13). The simulation employed a passband of 10 to 100 kHz. Upon impacting the undamaged rock block surface, the resulting phase and group velocity functions would appear as shown in figure 1. Since the dynamic properties of each layer govern the dispersion, propagation of each Rayleigh wave to successively deeper layers results in a constant phase and group velocity over

the complete spectrum for the uniform layer case. The Rayleigh wave phase velocity is computed to be approximately nine-tenths of the shear-wave velocity. Since these results are consistent with that calculated analytically (8), the computer program was assumed to be operating correctly. The uniform layer model, however, is unrealistic for simulation of mine structures, such as a mine pillar, rib, or roof. Fragmentation damage, stress relief, and zones of stress concentration normally cause a nonuniform distribution of elastic properties away from the edge of the openings.

### DAMAGED ROCK MODEL

To establish a physical model for the effects of induced damage or stress relief, simulating damage at the free surface of a mine opening, surface damage was induced in a Barre granite block by subjecting it to infrared heating. The block was heated for 16 min, then cores were taken from the granite block and evaluated using a diametric pulse velocity technique (14) at 2.54-cm depth intervals along the length of each cylinder. A graph of the compressional wave velocity profile as a function of depth is given in figure 2. The relative magnitude and depth of induced damage is reflected by reduced compressional wave velocity in the profile. A maximum heat energy density of  $8.1 \times 10^3$  J/cm<sup>2</sup> was produced at the surface. This corresponded to the maximum observed damage, which decreased more or less linearly with depth from the surface to a maximum depth of about 10 cm. The maximum amount of damage corresponded to roughly a 10 pct decrease in compressional wave velocity, with respect to that of the intact (undisturbed) material. The approximately

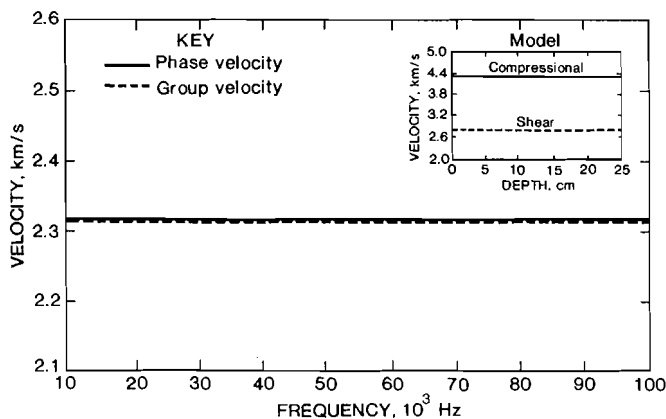


Figure 1.—Rayleigh wave dispersion in undamaged Barre granite block.

constant velocity beyond the 10 cm depth reflects the uniformity of the undamaged, intact rock.

The Rayleigh wave response for propagation across the surface of a damaged granite block was simulated using the compressional wave and corresponding shear-wave velocity profiles, assuming a constant compressional to shear-wave velocity ratio of 1.53. The velocity profiles are partitioned into 1.25-cm intervals and combined with density ( $2.64 \text{ g/cm}^3$ ) for input into the forward Rayleigh wave dispersion model shown in figure 3. The simulation considers a passband of 10 to 100 kHz, the frequency range of most interest for fracture detection in rock of this type.

The utility of the Rayleigh wave dispersion analysis for detecting damage in rock is demonstrated by comparing the dispersion characteristics before (fig. 1) and after (fig. 3) heating of the granite block. The theoretical Rayleigh wave response for an impact on the damaged surface shows that phase and group velocity functions both are nonlinear and exponentially decrease with increasing frequency. Phase velocity remains higher in magnitude than group velocity, but asymptotically approaches nearly the same velocity beyond 100 kHz. This Rayleigh wave behavior typifies the normal dispersion associated with structures having an increase in dynamic properties with depth. While the phase velocity at 10 kHz can be attributed to the intact granite material, it is not readily apparent as to where the transition from damaged to undamaged material occurs.

The amplitude of Rayleigh wave particle motion is maximum at about one wavelength and decreases exponentially with depth below the surface. Hence, the influence of the material properties on the propagation of the Rayleigh wave is constrained essentially to a depth of approximately one wavelength (9). For this reason, plots of the dispersion parameters as a function of wavelength offer a method by which the zone of rock material properties affected by stress relief (or fragmentation damage) can be detected (fig. 4). For wavelengths shorter than the total depth of damage, or in our case the 10 cm depth of damage from heat penetration, the Rayleigh wave velocity is controlled primarily by the dynamic properties of the damaged rock. Beyond this depth, the Rayleigh wave velocity is primarily a function of the dynamic properties of the intact rock. Figure 4 shows that the calculated phase velocity in the damaged rock block remains nearly linear in the damaged zone and reaches a minimum at the surface, coincident with the interval of maximum induced damage. At this point, the phase velocity corresponds to a factor of 0.9 times the shear-wave velocity. As the Rayleigh wave propagates to greater depths within the damaged zone, the phase velocity increases in a linear fashion to a depth of about 10 cm, reflecting an increase

in the dynamic material properties, and signifying decreasing damage with depth. Beyond a depth of 10 cm,

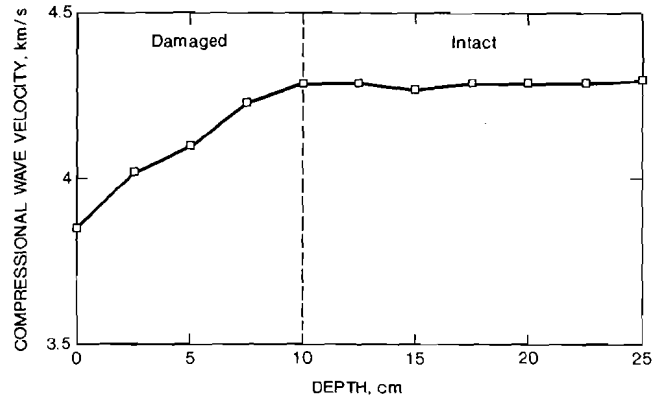


Figure 2.—Compressional wave velocity profile for damaged (heat induced) Barre granite block.

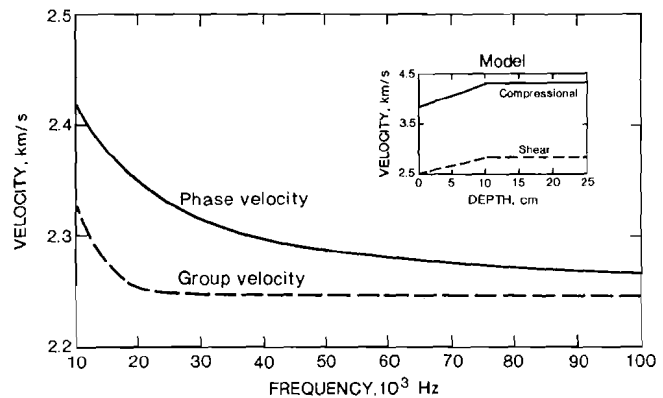


Figure 3.—Rayleigh wave dispersion in damaged Barre granite block.

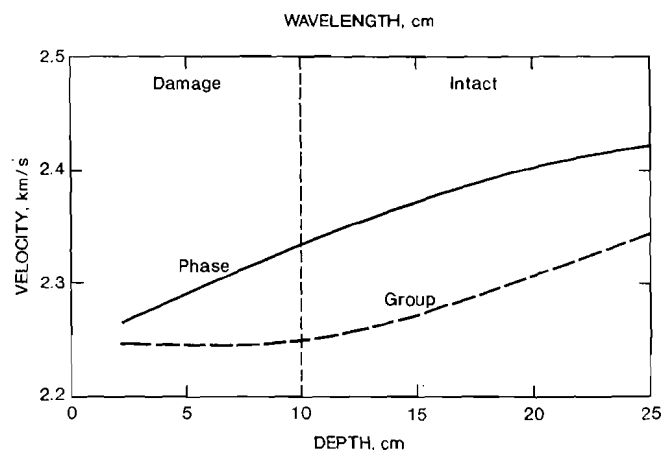


Figure 4.—Phase and group velocity as function of wavelength for damaged Barre granite block.

the phase velocity is nonlinear and the slope decreases (flattens out) for correspondingly longer wavelengths that penetrate the zone of undamaged material. Ultimately, the phase velocity reaches a maximum velocity (with a factor of about 0.85 times the intact shear-wave velocity), at wavelengths by more than roughly 30 cm. The loss of resolving power with depth contributes to the observation that the phase velocity does not immediately reach a constant value at the maximum depth of damage (10 cm).

Since the group velocity depends primarily on the change in phase velocity with frequency, it can be a more

sensitive dispersion parameter than its counterpart phase velocity. Figure 4 reveals that the group velocity remains constant with increasing penetration of the Rayleigh wave to a depth of 10 cm, the maximum depth of damage. Beyond this point, the group velocity increases ultimately approaching the phase velocity about 40 cm. In contrast to the phase velocity, the change in group velocity beyond the damaged zone appears to offer a better indicator for distinguishing the damaged from the undisturbed (intact) rock zone.

## RAYLEIGH WAVE DISPERSION IN MINE STRUCTURES

Investigations using sonic methods near mine workings by others (2-3) have demonstrated that elastic waves change in character away from the mine openings, defining three stress (damage related) zones. The boundaries of each zone are influenced by the size and shape of the workings, the type and depth of mechanical excavation, damage and stress relief, mechanical properties of the rock, and the natural state of stress. The generalized velocity profile adjacent to mine workings defines three stress zones; (1) a stress-relieved zone; (2) a stress-concentration zone; and (3) a virgin-stress zone (fig. 5). The first zone, attributed to stress relief, generally is a lower velocity zone for body waves, increasing inward. Wave velocities are below those associated with the undisturbed material (4). In the second zone, the velocity increases at a faster rate reaching a maximum value. This behavior relates to a zone of stress concentration, where the maximum velocity coincides with the peak stress occurring in the mine structure. Beyond this maximum velocity, velocity decreases to a more constant, background

velocity, representing that for the undisturbed material. The variation between minimum velocity, associated with the stress-relieved zone, and the maximum velocity, associated with the peak stress zone, has been reported as high as 60 pct for measurements in coal pillars (3).

### COAL MINE PILLAR MODEL

The shear-wave velocity profile obtained from sonic measurements taken in a borehole penetrating a coal mine pillar (3) was used to derive a Rayleigh wave model for coal mine support structures. Compressional wave velocity was estimated, based on the shear-wave profile, and in combination with the bulk density for coal was used to calculate a dynamic elastic moduli data set for the stress relieved and stressed portions of the mine pillar as input into the forward model. Ten velocity intervals are used; 9 at 0.3 m in thickness, and the last representing a semi-infinite half-space. A compressional to shear-wave velocity ratio of 1.6 was assumed in the calculation of compressional wave velocity. Since density is known to have only a second-order effect (15) on Rayleigh wave propagation, a uniform coal density of 1.75 g/cm<sup>3</sup> is assumed. The velocity dispersion is shown in figure 6 together with a schematic depicting the dynamic model used. In contrast to the earlier models, the calculated Rayleigh wave phase and group velocities cross at a frequency of about 200 Hz, and tend to decrease with an increase in depth of penetration to asymptotic values. Both velocities are nonlinear and appear to converge to a nearly constant value beyond 1 kHz.

Figure 7 gives the theoretical phase and group velocities plotted as a function of wavelength for a Rayleigh wave propagated along a stressed coal mine pillar, assuming the conditions outlined above. Rayleigh wave dispersion at increasing distances from the pillar surface appears to define the three stress zones; i.e., changes in the continuity of the phase and group velocities reflect a stress relief, stress concentration, and virgin stress zone.

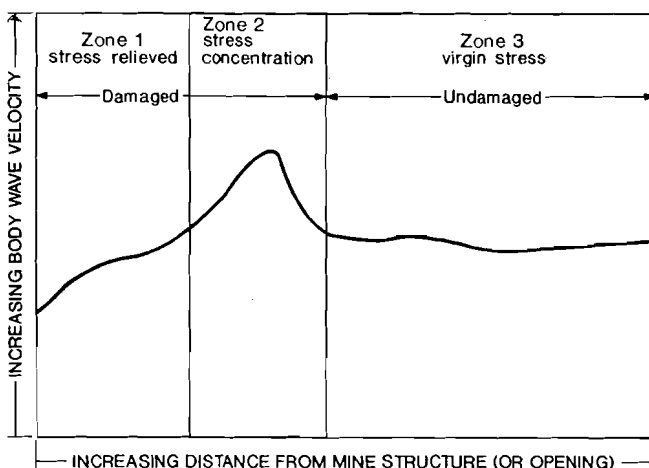


Figure 5.—Generalized velocity (bodywave) - stress (or damage) relationship as function of distance from mine structure (or opening).

The phase velocity increases from a low in the stress-relieved zone to a high in the peak stress zone. Beginning at the boundary between the peak and virgin stress zones, the phase velocity flattens out becoming nearly constant. The constant velocity shows the uniform dynamic properties (particularly the shear-wave velocity) associated with the virgin stress, or undisturbed region. Also of importance, the fact that the group velocity minimum appears to mark the approximate midpoint between the pillar face and the beginning of the virgin stress zone. This relationship provides a generalized rule-of-thumb, whereby the distance from the pillar face to the virgin stress zone can be approximated by multiplying the wavelength where group velocity is a minimum by two. In the pillar model, the group velocity minimum occurs at a wavelength of roughly 1.3 m; hence, the distance to the virgin stress zone would be estimated to be approximately 2.6 m. This is in good agreement with the actual distance of 2.7 m used

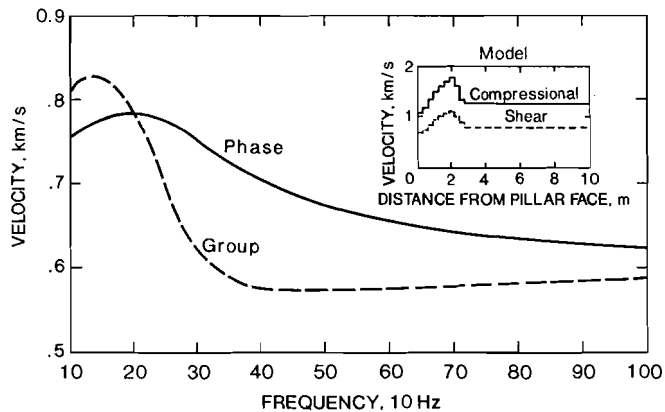


Figure 6.—Rayleigh wave dispersion for stressed coal pillar model.

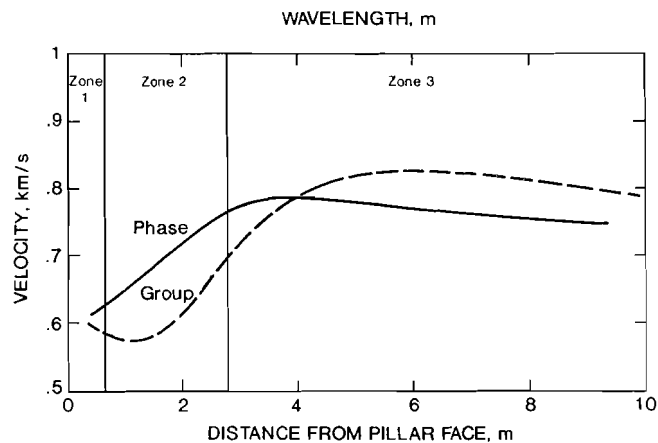


Figure 7.—Phase and group velocity as function of wavelength for stressed coal pillar model.

in the model. Beyond the group velocity minimum, the velocity increases and begins to flatten out in the undisturbed virgin stress region (beyond about 4 m). For this case, the group velocity reaches a value slightly greater than the shear-wave velocity, in the virgin stress region.

The three zones of stress can be depicted more clearly from a graph where the group velocity is subtracted from the phase velocity. Examination of figure 8 reveals that the basic shape of the difference function correlates well with the body wave profile of figure 3, suggesting three distinct zones of stress. The boundaries between each zone can be approximately located by noting the inflection points on either side of the peak velocity difference and their corresponding wavelengths. More importantly, the peak velocity difference gives an estimate of the distance to the peak stress in the coal pillar. For the coal mine pillar simulation, the peak difference occurs at a wavelength of about 1.9 m, corresponding to a distance of 1.9 m to the peak stress zone used in the model. Knowledge of the depth to peak stress is useful for optimum anchor placement, and/or for implementation of stress relief procedures.

To be able to utilize Rayleigh wave dispersion technology for the assessment of damage and stress relief in mine structures, data must be collected using a calibrated system, and a source signature bandwidth that is mutually compatible with the transducer and computer response. Otherwise, important dispersion information may be selectively attenuated. Another consideration for calibration is to ensure that the source signature bandwidth is capable of generating Rayleigh wavelengths that are suitable for penetrating to the depths of interest. The basis for appropriate system calibration can be provided through forward modeling, with appropriate graphical representation of the Rayleigh wave dispersion.

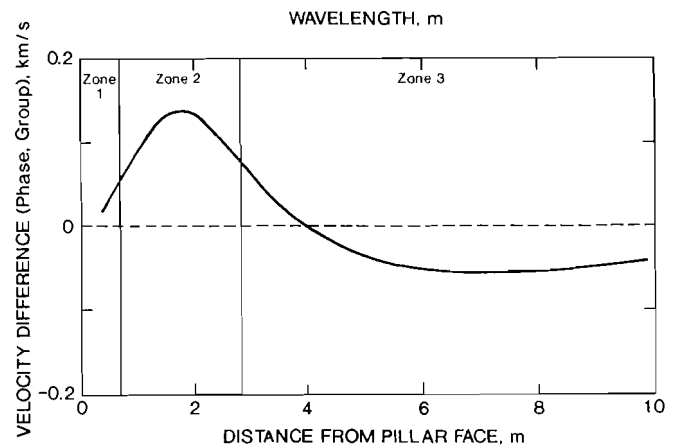


Figure 8.—Velocity difference (phase-group) as function of wavelength for stressed coal pillar model.

The theoretical wavelength-frequency relationship for Rayleigh wave propagation in a stressed coal mine pillar is shown in figure 9. Superimposing the three zones of stress with the group or phase dispersion response indicates the required bandwidth for coal mine pillar investigations. Low-frequency, long-wavelength surface waves penetrate to greater depths into the virgin stress zone, whereas high-frequency, low-velocity waves show the stress-relieved, fracture zone. The stress concentration zone is defined by intermediate wave frequencies. These results illustrate the need for a broadband source of 100 Hz to 1 kHz to investigate or resolve the different stress zones in coal. Moreover, the impulse response of both the accelerometer and computer should be sufficiently broadband and compatible with that anticipated to be useful for investigating the problem at hand.

### METAL MINE ROOF MODEL

The physical model used to evaluate dispersion characteristics resulting from Rayleigh wave propagation for the hypothetical case of a damaged metal mine roof is based on a field-derived, conventional body-wave velocity profiles, obtained from sonic logging (4). The compressional and shear-wave data are combined with a bulk density ( $2.6 \text{ kg/m}^3$ ) data for input into the forward model. For this case, 30 velocity intervals are used. The first interval is 0.3 m thick, the second through twenty-ninth are 0.1 m thick, and the last represents a semi-infinite half-space.

The phase and group velocity functions, i.e., upon impacting the damaged roof rock, assuming a source response function between 500 Hz to 5 kHz, are calculated as shown in figure 10. Figure 10 also gives the shear and compressional wave data used in the dynamic model. Both velocity functions are nonlinear and appear to converge asymptotically to a nearly constant value beyond 5 kHz.

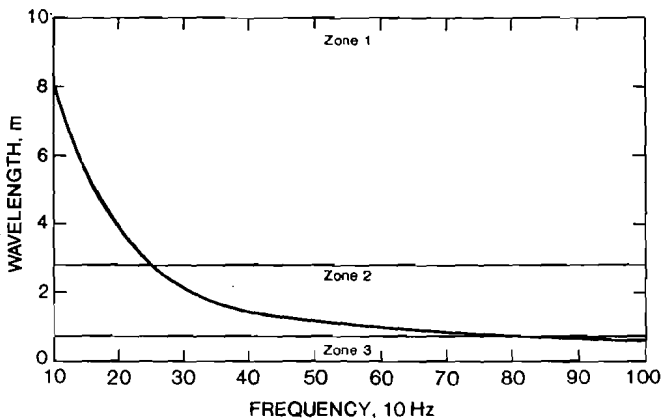


Figure 9.—Wavelength as function of frequency for stressed coal pillar model.

Figure 11 gives theoretical phase and group velocities plotted as a function of wavelength for a Rayleigh wave propagated along a stressed metal mine roof. The calculated Rayleigh wave phase and group velocities for the mine roof tend to decrease with a decrease in depth of penetration (higher frequencies). The Rayleigh wave dispersion at progressively increasing distances from the mine roof appears to define three stress zones and is qualitatively similar to the dispersion results for the coal mine pillar (fig. 7). Changes in the continuity of the phase and group velocities again reflect a stress relief, stress concentration, and virgin stress zone. The phase velocity increases from a low in the stress-relieved zone and reaches a constant value at a wavelength of about 5 m in the virgin stress zone. The tendency toward approaching a constant velocity with depth shows the more uniform dynamic properties and stress condition in the undisturbed region.

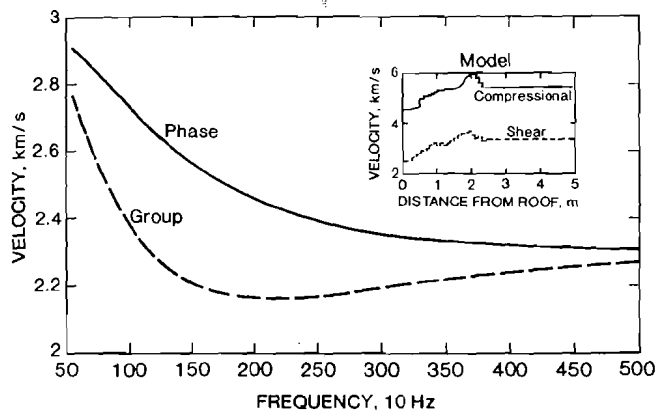


Figure 10.—Rayleigh wave dispersion for stressed metal mine roof model.

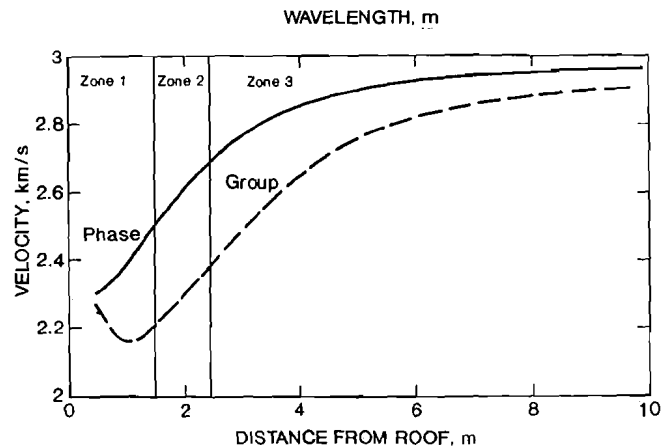


Figure 11.—Phase and group velocity as function of wavelength for stressed metal mine roof model.

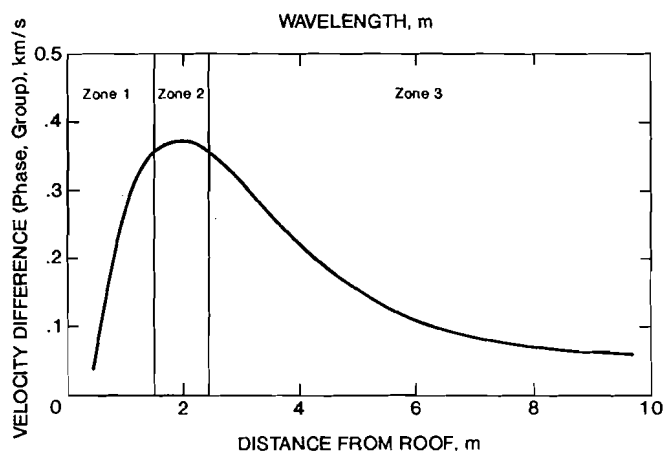


Figure 12.—Velocity difference (phase-group) as function of wavelength for stressed metal mine roof model.

Again, the group velocity minimum at a wavelength (depth) of about 1.1 m appears to mark the approximate midpoint between the pillar face and boundary of the virgin stress at 1.9 m. The group velocity then increases and begins to flatten out in the undisturbed virgin stress region, but does not become constant until the wavelength exceeds 6 m. In general, the rate decrease in phase and group velocity also begins in the peak stress zone, but it is not as rapid as in the coal mine pillar.

The phase minus the group velocity again makes the three zones of stress more easily identified (fig. 12). As with the coal pillar model, this plot correlates well with the body wave profile (fig. 8), indicating the three stress zones. Moreover, the peak velocity difference appears to locate

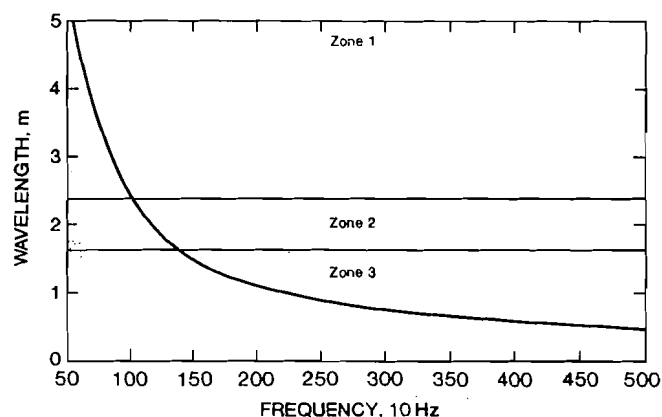


Figure 13.—Wavelength as function of frequency for stressed metal mine roof model.

the peak stress for this metal mine roof about 1.9 m. The difference function has steeply increasing slope in the stress-relieved zone, nearly flat slope at peak velocity difference in the stress concentration zone and a decreasing slope to asymptotic, low-velocity difference in the virgin stress zone.

The theoretical group wavelength-frequency relationship for Rayleigh wave propagation along the metal mine roof is shown in figure 13. Superimposing the three established stress zones over the exponentially decaying dispersion response gives an indication of bandwidth requirements for Rayleigh wave investigation of metal mine structures. These results suggest the need for a broadband source function of at least 500 Hz to 2.5 kHz to adequately resolve the three expected stress zones near a mine working.

## RAYLEIGH WAVE ATTENUATION: NOVEL DISPERSION PARAMETER

In addition to the more conventional Rayleigh wave velocity modeling described earlier, attenuation was examined as a novel dispersion parameter to more fully exploit Rayleigh wave information concerning rock properties. The derivation of the attenuation dispersion is based on a convolutional Rayleigh wave model expressed by

$$r(t,x) = s(t) * e(t,x) * i(t), \quad (4)$$

$$r(t,x) \leftrightarrow R(\omega,x), \quad (5)$$

$$R(\omega,x) = S(\omega)E(\omega,x)I(\omega), \quad (6)$$

where  $r,R$  = Rayleigh wave (time, ms and frequency, Hz, respectively),

$s,S$  = source signature (time, ms and frequency, Hz, respectively),

$e,E$  = Earth filter (time, ms and frequency, Hz, respectively),

$i,I$  = instrumentation filter (time, ms and frequency, Hz, respectively),

$t$  = time, ms,

$x$  = distance, m,

$\leftrightarrow$  = Fourier transformation,

and  $*$  = convolution operator.

Assuming the experimental configuration uses a single source, and simultaneous recordings at two identical transducers, a spectral ratio can be expressed by

$$\frac{R_2(\omega, x_2)}{R_1(\omega, x_1)} = \frac{|E(\omega, x_2)| \exp j \theta_2(\omega)}{|E(\omega, x_1)| \exp j \theta_1(\omega)}, \quad (7)$$

$$A(\omega, x) = |A(\omega, x)| \exp j \phi(\omega), \quad (8)$$

where  $|E(\omega, x)|$  = amplitude spectrum of the Earth filter,

$A(\omega, x)$  = Rayleigh wave spectral ratio,

$|A(\omega, x)|$  = amplitude spectrum of  $A(\omega, x)$ ,

$x$  = distance from seismic source,

$\theta(\omega)$  = phase angle of Rayleigh wave,

$\phi(\omega)$  = phase spectral ratio,

1 = subscript refers to near transducer,

2 = subscript refers to far transducer,

and  $j$  = complex number with real and imaginary parts.

Assuming the Rayleigh wave propagation in rock to be

$$E(\omega, x) = \left[ \left[ \frac{E(\omega, x_0)}{x^{1/2}} \right] \exp j (-\alpha(\omega) x + kx) \right], \quad (9)$$

where  $E(\omega, x_0)$  = amplitude of Earth filter at source,

$x_0$  = source location,

$\exp j (-\alpha(\omega)x + kx)$  = energy loss due to absorption,

$k$  = wavenumber,

$\alpha(\omega)$  = Rayleigh wave attenuation coefficient,

and  $x^{-1/2}$  = energy loss due to spherical divergence.

Substituting equation 9 into the convolutional model, gives the resultant spectral ratio

$$A(\omega) = \left[ \frac{x_2}{x_1} \right]^{1/2} \exp j (-\alpha(\omega) \delta x + k \delta x). \quad (10)$$

Equating equations 8 and 10 and solving for the Rayleigh wave attenuation coefficient then gives the expression,

$$\alpha(\omega) = -\frac{1}{\delta x} \ln \left[ \frac{x_2^{1/2}}{x_1} |A(\omega)| \right]. \quad (11)$$

Converting from units of nepers to decibels per centimeters gives,

$$\alpha(\omega) = -\frac{20}{\delta x} \log_{10} \left[ \left[ \frac{x_2}{x_1} \right]^{1/2} |A(\omega)| \right]. \quad (12)$$

This Rayleigh wave attenuation function provides an uncomplicated expression in terms of the distances from the source and amplitude spectral ratio of the recorded Rayleigh waveforms. To gain insight into attenuation changes with depth, however, the attenuation is expressed as a function of wavelength. By using the relationship given in equation 1, the Rayleigh wave attenuation function becomes

$$\alpha(\lambda) = -\frac{20}{\delta x} \log_{10} \left[ \frac{x_2^{1/2}}{x_1} |A(\lambda)| \right]. \quad (13)$$

This equation is the basis for a dispersive attenuation function in a computer program termed RAYDAN (RAYleigh wave Damage ANalysis) can be used to assess the depth of damage or, correspondingly, the effects of stress at depth in mine structures, based on changes in the dynamic material properties of the rock. The RAYDAN program therefore provides another tool utilizing attenuation, in addition to velocity dispersion, for interpreting damage or stress conditions in mine structures.



## INVERSE MODELING

It is anticipated that once a prototype system is developed for use in the mine environment, a more comprehensive and accurate assessment of damage from overbreak and stress relief in mine rock can be obtained using a Rayleigh wave inversion process. In seismology, for example, characteristics of earthquake generated Rayleigh waves are inverted to interpret geologic characteristics of the Earth's mantle (16-17) and crust (18). In more recent applications, the Rayleigh wave inversion approach was used to determine pavement thickness (19) and for geotechnical site characterization (20-21). A prerequisite for inverse modeling is the calculation of the phase and/or group velocity functions from recorded Rayleigh waves. The computer program called RAYDAN accomplishes this task using Fourier analysis. The RAYDAN phase velocity relationship,

$$V_p(\omega) = \frac{\omega \delta x}{\phi(\omega) \pm 2\pi n}, \quad (14)$$

was derived by equating the phase spectra given in equations 8 and 10 and rearranging the terms. The group

velocity is then computed, based on determination of the phase velocity between two receivers and the relationship given by equation 3.

The inverse approach requires a physically realistic model, using the forward solution together with the observed dispersion relationships. For instance, a "best guess" trial model must first be used to generate a theoretical Rayleigh wave dispersion curve. Since higher frequencies (shorter wavelengths) penetrate to lesser depths and resolution decreases with depth, this process begins by perturbing first the dynamic properties associated with the surface layer. The updated theoretical dispersion functions can then be plotted and compared with those observed in the field. This process is repeated one layer at a time until good agreement exists between both the modeled and field-derived dispersion plots. The observation that only the shear-wave velocity exerts a first-order influence on Rayleigh wave propagation, suggests that the other dynamic constants of compressional wave velocity and density can, for a first approximation, be considered to remain constant (21).

## CONCLUSIONS

Various models were investigated for the use of Rayleigh wave velocity and attenuation dispersion to detect and delineate damage and stress relief zones in mine structures. The models incorporated data from a thermally damaged rock block and from field observations of velocity change in body waves in a coal mine pillar or metal mine roof. It was found that the velocity dispersion occurred at the higher frequencies (shorter wavelengths) in a rock block damaged by heating with microwaves. Additionally, it was found that the group velocity minimum and phase and group velocity cross points occurred near the boundaries of the stress-relieved and stress-concentration zones for a modeled coal mine pillar, associated with a wavelength slightly greater than the expected depth. Similar results generally depicting distinct stress zones near underground workings for the metal mine roof case. A computer program called RAYDAN

was developed for applying frequency dependent attenuation to the detection of damaged or stress-relieved zones in mine pillars. The results of modeling investigations provide encouragement for the use of Rayleigh wave velocity and attenuation dispersion characteristics in investigation of the integrity of mine structures, particularly when coupled with inverse modeling. The theoretical and experimental modeling suggests that a system with compatible source, transducers, computer can be assembled with existing hardware, and can be used for the detection of stressed or damaged zones in mine structures. Follow-on efforts will assemble and field test such instrumentation. Recommendations are made that the Rayleigh wave frequency-dependent, attenuation program be further refined and that verification tests be performed at minesites, following development of surface wave generating and receiving field instrumentation.

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