EVALUATION OF GAGES FOR MEASURING DISPLACEMENT, VELOCITY, AND ACCELERATION OF SEISMIC PULSES

BY B. E. BLAIR AND W. I. DUVALL

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LIST OF SYMBOLS

a = Peak particle acceleration.

c = Propagation velocity in rock medium.

D = Distance between the gage and the explosive.

K = Intercept for empirical equations.

n = Slope for empirical equations.

P = Pressure at cavity wall.

$\bar{r}$ = Scale parameter.

ρ = Density of rock.

$\bar{\sigma}$ = Standard deviations.

t = Time.

u = Peak particle displacement.

v = Peak particle velocity.

V = Volume of charge.

mv = Volts x 10^{-3}.

μv = Volts x 10^{-6}.

Ω = Ohms.

$\mu\Omega$ = Ohms x 10^{-6}.

g = Acceleration owing to gravity

SUMMARY

Accelerometers of various types, velocity gages, and a displacement meter are shown to give reliable data when measuring seismic pulses generated in rock by the detonation of explosive charges. Displacement and velocity records are differentiated and shown to compare with velocity and acceleration records. Velocity and acceleration records are integrated and shown to compare with displacement and velocity records, respectively. The experimental data are shown to satisfy the scaling laws developed by dimensional analysis. Propagation laws are developed for peak amplitudes of the first pulse of displacement, velocity, and acceleration. These propagation laws are shown to be independent of the gage used to obtain the data.

As a result, the usefulness of the various gage types can be extended. For example, accelerometers, because of their high frequency and amplitude limits, can be employed near the shot to measure acceleration and/or velocity (by integration). Velocity gages, because of their higher sensitivity, can be employed at relatively larger distances from the shot to measure velocity and displacement or acceleration (by integration or differentiation).

INTRODUCTION

Seismic pulses, generated by detonating explosive charges in rock, are usually studied by measuring one or more of the quantities - particle displacement, velocity,[3/] and acceleration. These quantities are related both time wise and distance wise. The functions relating these quantities are useful for showing that gage records are true representations of the particle motion of the rock to which they are anchored. Thus, for gages at the same point, velocity and acceleration records should agree with differentiated displacement and velocity records, and displacement and velocity records should agree with integrated velocity and acceleration records.[4/] Furthermore, the decay of peak amplitude with distance for displacement, velocity, and acceleration should be independent of the gage used. Other investigators[5/][6/] have shown that displacement, velocity, and acceleration can be differentiated and/or integrated on seismic records of low frequency and long duration. This paper shows that these same operations can be performed, within the accuracy of the original measurements, on seismic records of relatively high frequency and short duration.

3/ Velocity is used to designate particle velocity. Propagation velocity will be used to designate the rate at which the wave travels.

4/ Displacement and acceleration records can be differentiated or integrated twice to obtain acceleration and displacement, respectively. However, the error involved is large.

5/ Neumann, Frank, An Appraisal of Numerical Integration Methods As Applied to Strong Motion Data: Bull. Seism. Soc. Am., vol. 33, 1943, pp. 21-60.

6/ Ruge, Arthur C., Discussion of Principal Results from the Engineering Viewpoint: Bull. Seism. Soc. Am., vol. 33, 1943, pp. 13-20.

The cost in time and money and the availability of recording equipment limits the number of measurements that can be made for any particular seismic investigation. Thus, an efficient use of gages and recording channels is necessary. By showing that the various gage records can be differentiated and/or integrated, the number of measurements of displacement, velocity, and acceleration can be increased two- or three-fold over the number of gages or recording channels used. Also, by using gages with high frequency and amplitude limits close to the shot point, and gages with high sensitivity and low frequency response far from the shot point, the distance range for measurements can be extended over that obtained from any one type of gage.

ACKNOWLEDGMENTS

The field tests at the Fort Randall and Oahe dam sites, S. Dak., were made in cooperation with the Corps of Engineers, Omaha District. Ralph Folkenroth and Lawrence Gray assisted in the field measurements.

EXPERIMENTAL PROCEDURE - INSTRUMENTATION

Briefly, the experimental procedure was as follows: (a) Instruments for measuring displacement, velocity, and acceleration were mounted on the rock surface at distances varying from 26 to 1,000 feet from the shot. (b) Single charges varying from 6.25 to 100[7] pounds of Hercomite B explosive were placed in 4-1/2-inch-diameter drill holes 20 feet deep, stemmed to the surface with shale cuttings, and detonated with primacord. (c) Seismic records were obtained for 10 shots in Pierre shale at the Oahe dam site. In addition several records were obtained from 2 shots (1 single and 1 millisecond delay) in Niobrara chalk at the Fort Randall dam site where gages of various types were mounted at the same point to study the reproducibility of results.

The following gages were employed in this series of tests: a Leet displacement meter, MB velocity gages, and Statham, General Electric, and Gulton accelerometers. The gages are pictured in figure 1 and their characteristics are summarized in table 1. The manufacturer's specified amplitude and frequency limits are shown in figures 2 and 3. Within the limits of the gage sensitivities and the restrictions imposed by the amplifier-recording system, the Gulton accelerometer is the most sensitive gage at frequencies above 400 c.p.s. - the MB velocity gage for frequencies under 400 c.p.s. The other gages, although of relatively lower sensitivity, have characteristics, such as high amplitude limits, low impedance, or high frequency response, which may make these more desirable for a particular application.

With the exception of the Leet displacement meter, all gages were secured to metal studs anchored in the rock surface and oriented to respond to vertical movement. Generally, the gage studs were cemented into surface holes about 6 inches deep with quick-setting gypsum cement. The smaller gages (MB and Gulton) also gave satisfactory results when attached to pointed studs driven several inches into surface holes. The acceleration and velocity gages were connected to the recording equipment with waterproof, shielded cable. The recording equipment, which was housed in an instrumentation truck, consisted of accessory amplifiers and two 8-channel, high-speed, cathode-ray recording cameras. A report describing similar equipment has been published.[8]

8/ Obert, Leonard, and Duvall, Wilbur I., A Gage and Recording Equipment for Measuring Dynamic Strain in Rock: Bureau of Mines Rept. of Investigations 4581, 1949, 12 pp.

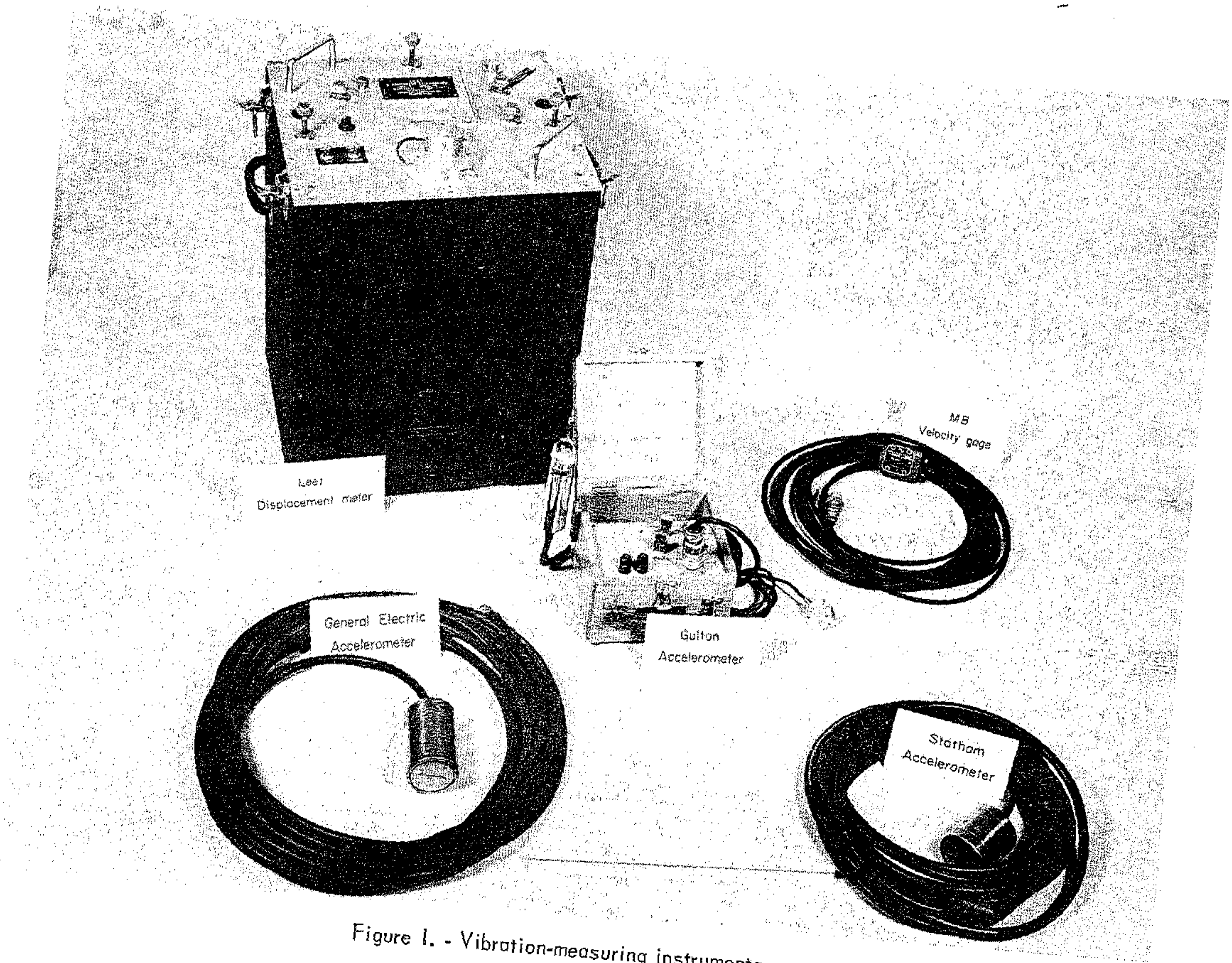


Figure I. - Vibration-measuring instruments.

TABLE 1. - Gage and amplifier characteristics

Name	Model No.	Transducer element	Gage natural frequency, c.p.s.	Gage sensitivity: Manufacturer's rating	Gage sensitivity: Effective value 1/	Limits of measurements 1/: Amplitude gage units	Limits of measurements 1/: Frequency, c.p.s.
Leet displacement meter	2-105	3-component mechanical-optical vibrating levers	1 damped to 0.6 critical	optical amplification factor - 50	Same	0.001-0.03 in. or 8.3-250 ft. x 10^{-5}	3-unspecified 2/
MB velocity gage	125	Electro-magnetic moving coil	4.75 undamped	96.3 mv/in./sec.	Same	0.005-8.0 in./sec. or 4.2-6,670 ft./sec. x 10^{-4}	10-2,000
Statham accelerometer	All-HV 1,000-1,500	Resistance wire-strain-gage bridge circuit	1,500 damped to 0.7 critical	$2.7 \mu \Omega/\Omega$/g	45μ v/g	3-1,000 g or 96.6-32,200 ft./sec.^{2}	10-1,000
General Electric accelerometer	9345002G1 3,214-405	Piezoelectric APD crystal	20,000 undamped	5.84 $\mu\mu$ coulomb/g	0.29 mv/g	10-5,000 g or 322-161,000 ft./sec.^{2}	10-10,000
Gulton accelerometer	A-403	Piezoelectric Barium titanate crystal	10,000 undamped	14 mv/g	Same	0.02-24 g or 0.644-733 ft./sec.^{2}	10-4,000

1/ When used in conjunction with the amplifier system employed in these tests.
2/ Fixed paper speed of 4 inches per second limits resolution of frequencies above 100 c.p.s.

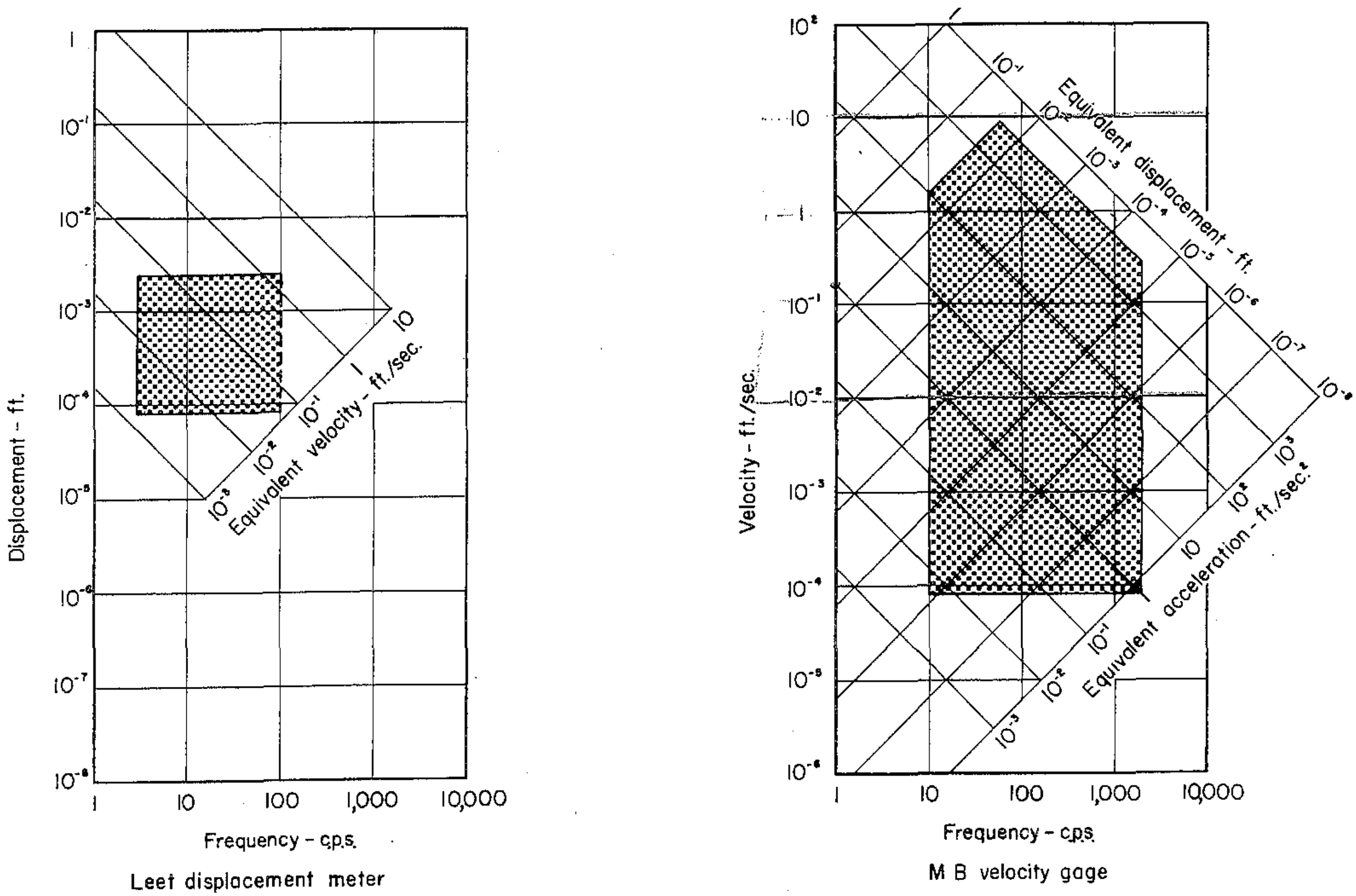


Figure 2. - Rated amplitude and frequency ranges of displacement meter and velocity gage.

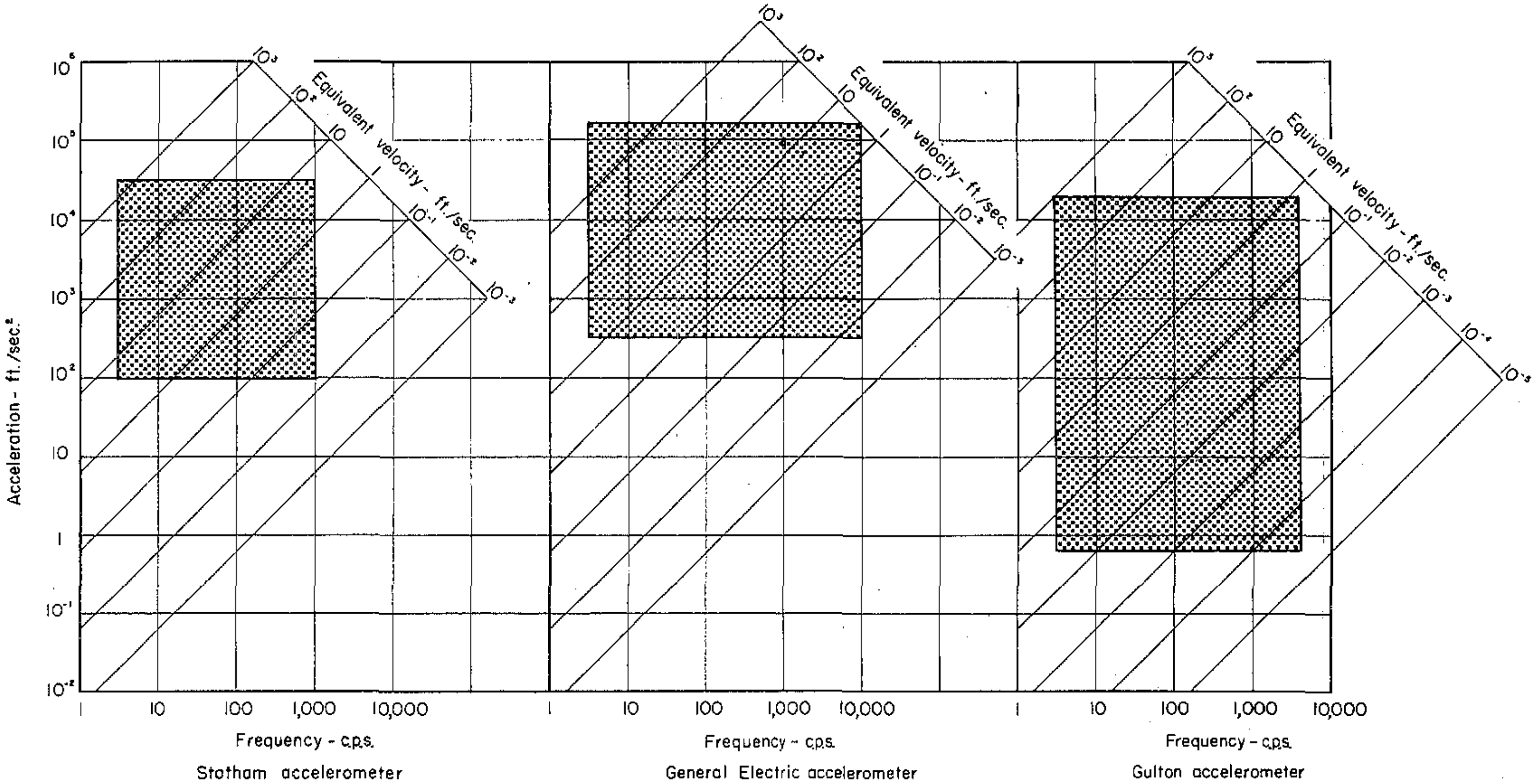


Figure 3. - Rated amplitude and frequency ranges of accelerometers.

The Leet displacement meter is a fixed-gain, self-contained instrument, which records transverse, vertical, and longitudinal components of displacement on 70 mm. photographic paper driven at a constant speed of 4 inches per second. This instrument was positioned and leveled at the desired surface test point; oriented with respect to the shot; and remotely operated from the instrumentation truck.

PRESENTATION AND INTERPRETATION OF DATA

Records from the Fort Randall tests are presented in figure 4 to show the reproducibility of the data. Records a, b, and c were obtained from instrumenting a single shot with three different types of accelerometers located at the same point. Records d and e were obtained from instrumenting a millisecond delay shot with two Gulton accelerometers at the same point. Records f and g are from two MB velocity gages located at the same distance from the millisecond delay shot. Comparable records show satisfactory agreement both in wave form and amplitude.

Directly measured and derived records are compared in figures 5 and 6. The derived records were obtained by graphical differentiation or integration at 2 millisecond intervals. In the graphical integration process the alignment of the base line is a major source of error. Some of the records contained an extraneous low frequency and, as the integral is inversely proportional to the frequency, a large shift in the derived wave base line was observed. This error is cumulative, and, although the error in the first peak is relatively small, it becomes increasingly larger for succeeding peaks. To make the integral curve approach zero for large times, it was often necessary to make a small adjustment in the alinement of the base line. Both the wave shape and magnitude of these records show relatively good agreement.

Because point-by-point differentiation and integration showed good agreement with observed records, only initial peak data were measured as follows:

1. From displacement records - directly measured initial peak displacement and graphically differentiated initial peak velocity.

2. From velocity records - directly measured initial peak velocity, graphically differentiated initial peak acceleration, and graphically integrated initial peak displacement.

3. From acceleration records - directly measured initial peak acceleration and graphically integrated initial peak velocity.

In addition predominant frequencies and travel times were determined from most records. Propagation velocities, required in the computation of reduced displacement, velocity, and acceleration, were determined from arrival time - travel distance curves - by the method of least squares, as shown in figure 7.

The displacement, velocity, and acceleration data are presented in tables 2, 3, and 4. The initial peak amplitudes for observed and derived records are grouped for a given charge size and distance. Also duplicate tests employing like charge sizes are grouped so that shots for a given distance can be compared. The same gage at a given distance for duplicate shots shows as much variation in peak amplitude as several types of gages at the same point from a given shot. In some cases, enough attenuation of the signal was not attained and, consequently, the record went off scale. The traces for these records were projected to their probable peaks, and the resulting values are shown in the tables as estimated.

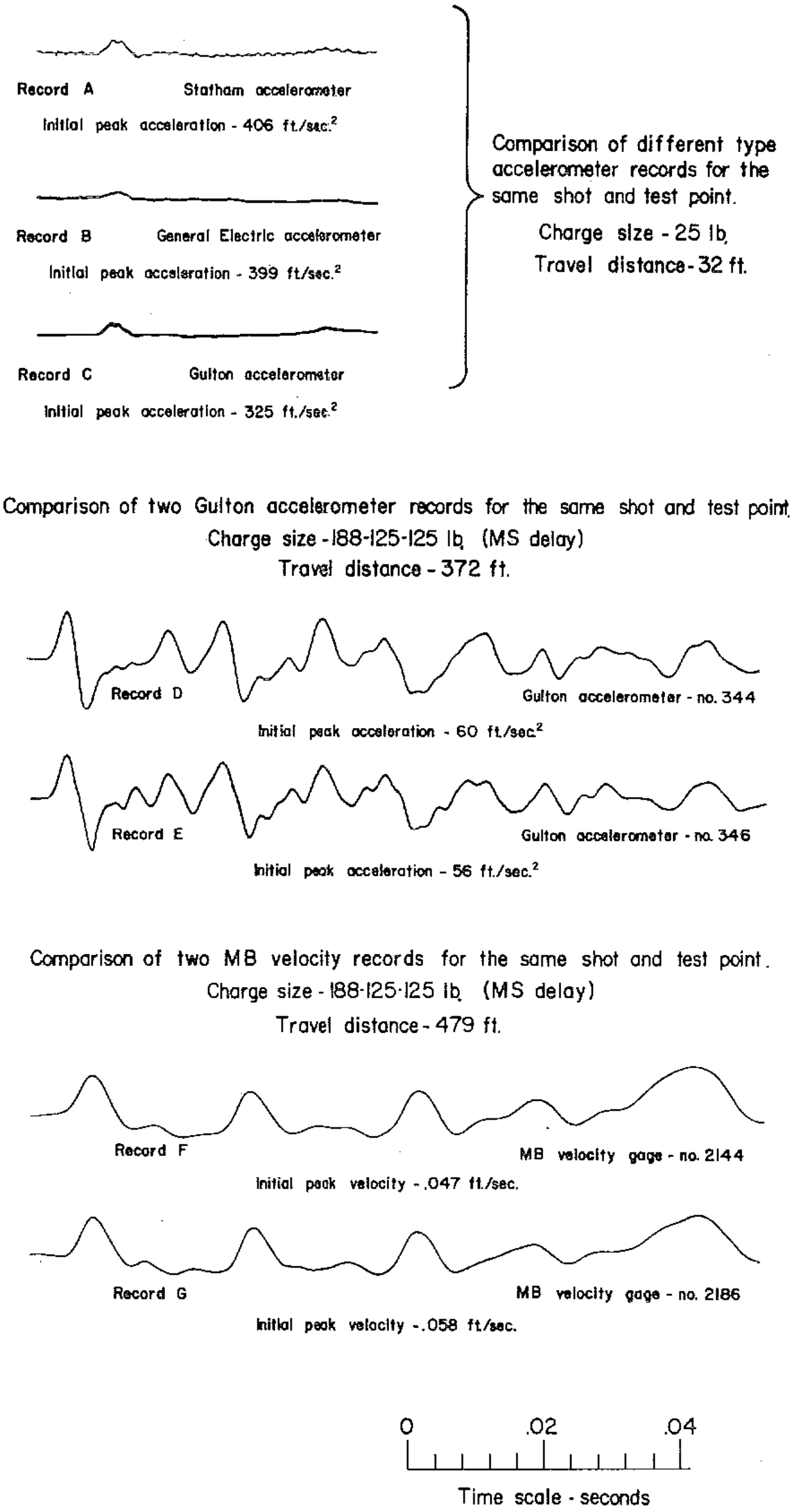
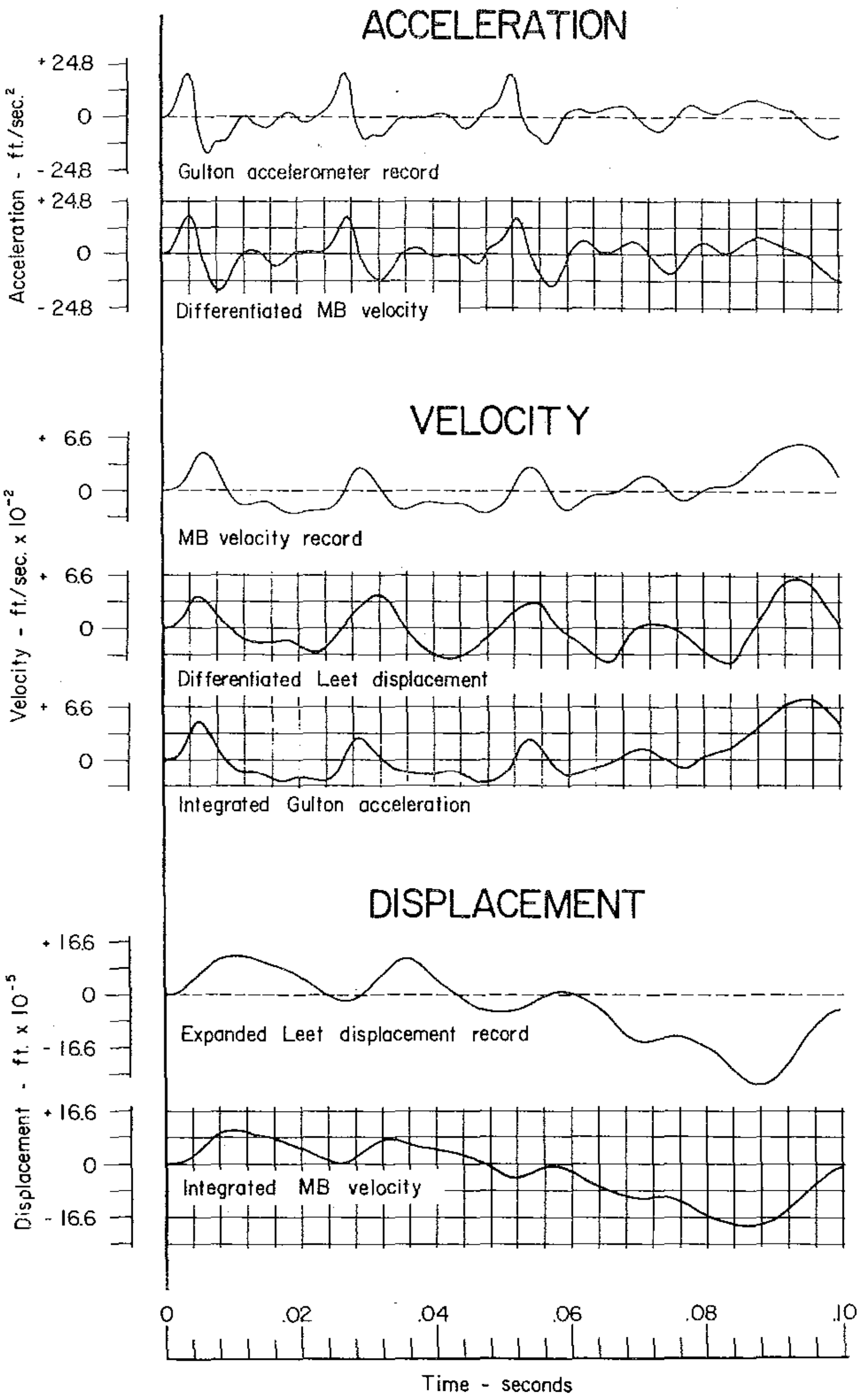


Figure 4. - Reproducibility of similar type gage records.



Charge size - 188-125-125 lb. (M.S. delay)
Travel distance - 479 ft.

Figure 5. - Comparison of derived and observed displacement, velocity, and acceleration—Fort Randall dam site.

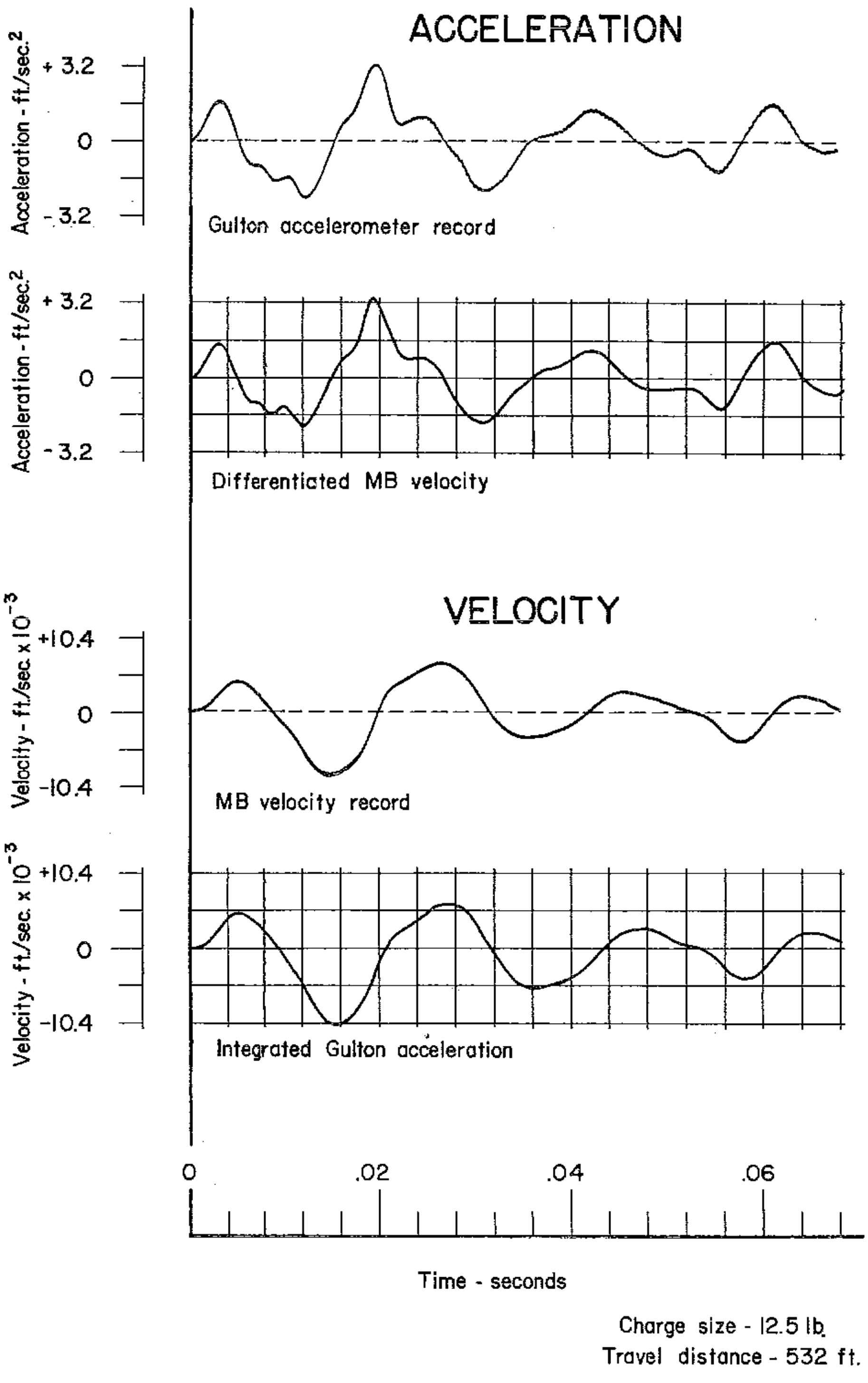


Figure 6. - Comparison of derived and observed velocity and acceleration—Oahe dam site.

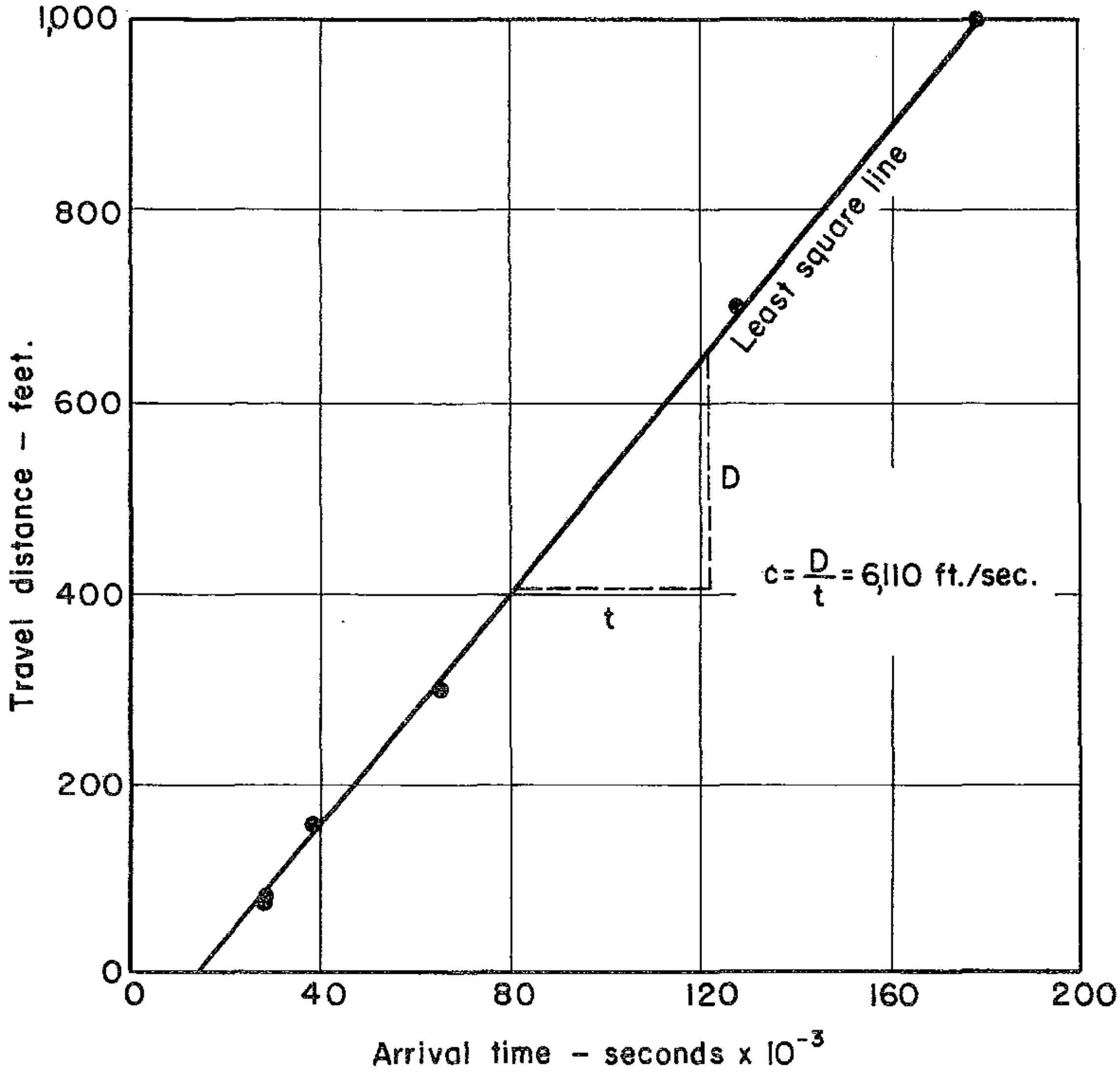


Figure 7. - Arrival time vs. travel distance, typical data for Pierre shale.

TABLE 2 - Displacement data in Pierre shale

Gage Type	Shot No.	Charge size, lb.	$\bar{r}$	Travel Distance, ft.	Travel velocity, ft./sec.	Initial vertical displacement, ft. x 10^{-5}	Initial pulse frequency, c.p.s.	$\frac{D}{\bar{r}}$	$\frac{u \rho c^2}{r}$, lb./ft.^{2}
Leet displacement.............	9	6.25	1.84	151	1/6,190	8.33	16	82.1	6,050
Integrated MB-velocity........				315		1.63	-	171	1,180
Do..................	1	12.5	2.32	78	5,990	101	-	33.6	54,500
Leet displacement.............				152		36.7	33	65.5	19,800
Integrated MB-velocity........				700		1.28	-	302	691
Do..................	6	12.5	2.32	78	5,720	55.1	-	33.6	27,100
Leet displacement.............				152		14.2	31	65.5	6,990
Integrated MB-velocity........				301		3.22	-	130	1,580
Do..................				532		2.01	-	229	989
Do..................	2	25	2.92	77.4	6,110	147	-	26.5	65,600
Leet displacement.............				151		46.7	28	51.7	20,800
Integrated MB-velocity........				300		2/11.2	-	103	2/5,000
Do..................	4	25	2.92	76	5,900	96.7	-	26.0	40,200
Leet displacement.............				150		25.0	20	51.4	10,400
Integrated MB-velocity........				300		6.21	-	103	2,580
Do..................				699		1.53	-	239	637
Do..................	3	37.5	3.35	77	6,100	2/288	-	23.0	2/112,000
Leet displacement.............				151		70.8	3/24	45.1	27,400
Integrated MB-velocity........				1,000		2.60	-	299	1,010
Do..................	5	37.5	3.35	76	5,850	80.2	-	22.7	28,600
Leet displacement.............				151		28.3	3/19	45.1	10,100
Integrated MB-velocity........				300		4.68	-	89.6	1,670
Do..................				549		2.84	-	164	1,010
Do..................	7	50	3.68	79	5,690	230	-	21.4	70,600
Leet displacement.............				151		66.7	17	41.0	20,500
Integrated MB-velocity........				312		11.6	-	84.8	3,560
Do..................	8	75	4.22	310	6,030	25.5	-	73.5	7,670
Leet displacement.............	10	100	4.64	150	5,730	90.0	3/13	32.3	22,200
Integrated MB-velocity........				300		36.2	-	64.7	8,940
Do..................				401		14.2	-	86.4	3,510

1/ Determined from gages used in conjunction with instrumentation truck. Arrival time can not be determined from Leet displacement records.

2/ Estimated.

3/ Higher frequencies present.

TABLE 3. - Velocity data in Pierre shale

Gage type	Shot No.	Charge size, lb.	$\bar{r}$	Travel distance, ft.	Travel velocity, ft./sec.	Initial vertical velocity, ft./sec. x 10-3	Initial pulse frequency, c.p.s.	$\frac{D}{\bar{r}}$	vρ c, lb./ft.^{2}
Integrated Gulton acceleration.......	9	6.25	1.84	27	6,190	1,270	-	14.7	27,400
Integrated Statham acceleration......				27		1,080	-	14.7	23,300
Integrated GE acceleration...........				27		900	-	14.7	19,400
Integrated Gulton acceleration.......				151		18.4	-	82.1	397
Differentiated Leet displacement.....				151		8.33	-	82.1	180
Integrated Gulton acceleration.......				292		4.98	-	161	108
MB velocity..........................				315		2.32	44	171	50.1
Integrated Statham acceleration......	1	12.5	2.32	28	5,990	1,710	-	12.1	35,700
Integrated Statham acceleration......				28		1,320	-	12.1	27,600
MB velocity..........................				78		182	47	33.6	3,800
Integrated Gulton acceleration.......				78		173	-	33.6	3,620
Do.				152		1/ 88.3	-	65.5	1/ 1,850
Differentiated Leet displacement.....				152		79.2	-	65.5	1,660
Integrated Gulton acceleration.......				300		11.9	-	129	249
MB velocity..........................				700		2.68	48	302	56.0
Integrated Statham acceleration......	6	12.5	2.32	28	5,720	1,830	-	12.1	36,500
Do..........................				28		1,640	-	12.1	32,700
MB velocity..........................				78		184	71	33.6	3,670
Integrated Gulton acceleration.......				78		170	-	33.6	3,390
Differentiated Leet displacement.....				152		23.3	-	65.5	465
Integrated Gulton acceleration.......				152		14.4	-	65.5	287
MB velocity..........................				301		5.18	47	130	103
Integrated Gulton acceleration.......				532		4.85	-	229	96.8
MB velocity..........................				532		4.29	58	229	85.6
MB velocity..........................	2	25	2.92	77.4	6,110	213	43	26.5	4,540
Integrated Gulton acceleration.......				77.4		175	-	26.5	3,730
Differentiated Leet displacement.....				151		58.3	-	51.7	1,240
Integrated Gulton acceleration.......				151		57.2	-	51.7	1,220
MB velocity..........................				300		1/ 17.9	42	103	1/ 382
Integrated Gulton acceleration.......				700		3.25	-	240	69.3
Integrated Statham acceleration......	4	25	2.92	25.6	5,900	1/ 3,110	-	8.77	1/ 64,000
Integrated GE acceleration...........				25.6		2,070	-	8.77	42,600
Integrated Statham acceleration......				25.2		1,930	-	8.77	39,700
MB velocity..........................				76		236	58	26.0	4,860
Integrated Gulton acceleration.......				76		211	-	26.0	4,340
Differentiated Leet displacement.....				150		53.3	-	51.4	1,100
Integrated Gulton acceleration.......				150		36.4	-	51.4	750
MB velocity..........................				300		8.50	42	103	175
Integrated Gulton acceleration.......				699		2.98	-	239	61.4
MB velocity..........................				699		2.93	56	239	60.3
Integrated Statham acceleration......	3	37.5	3.35	26.5	6,100	2,410	-	7.90	51,300
Integrated GE acceleration...........				26.5		1,670	-	7.90	35,600
MB velocity..........................				77		1/ 452	43	23.0	1/ 9,620
Differentiated Leet displacement.....				151		144	-	45.1	3,070
Integrated Gulton acceleration.......				151		62.4	-	45.1	1,330
Do..........................				700		4.52	-	209	96.2
MB velocity..........................				1,000		4.47	36	299	95.2
Integrated Statham acceleration......	5	37.5	3.35	25	5,850	2,980	-	7.46	60,800
Integrated GE acceleration...........				25		2,660	-	7.46	54,300
MB velocity..........................				76		227	57	22.7	4,630
Integrated Gulton acceleration.......				76		194	-	22.7	3,960
Differentiated Leet displacement.....				151		30.8	-	45.1	629
Integrated Gulton acceleration.......				151		28.0	-	45.1	572
MB velocity..........................				300		11.2	68	89.6	229
Do..........................				549		7.03	45	164	144
Integrated Gulton acceleration.......				549		6.03	-	164	123
Integrated GE acceleration...........	7	50	3.68	27.2	5,690	1,780	-	7.39	35,300
MB velocity..........................				79		469	36	21.4	9,310
Integrated Gulton acceleration.......				79		1/ 349	-	21.4	1/ 6,930
Do..........................				151		115	-	41.0	2,280
Differentiated Leet displacement.....				151		83.3	-	41.0	1,650
MB velocity..........................				312		13.1	34	84.8	260
Integrated Gulton acceleration.......				420		13.2	-	114	262
Integrated Statham acceleration......	8	75	4.22	26.3	6,030	1/ 6,780	-	6.23	1/ 143,000
Integrated GE acceleration...........				26.3		3,750	-	6.23	78,900
Integrated Gulton acceleration.......				77		735	-	18.2	15,500
Do..........................				151		71.3	-	35.8	1,500
MB velocity..........................				310		35.7	29	73.5	751
Integrated Statham acceleration......	10	100	4.64	25.4	5,730	1/ 10,800	-	5.47	1/ 216,000
Integrated GE acceleration...........				25.4		9,750	-	5.47	195,000
Integrated Gulton acceleration.......				76		647	-	16.3	12,900
Do..........................				150		82.8	-	32.3	1,660
Differentiated Leet displacement.....				150		43.3	-	32.3	866
MB velocity..........................				300		53.5	33	64.7	1,070
Do..........................				401		27.4	50	86.4	548

1/ Estimated.

TABLE 4. - Acceleration data in Pierre shale

Gage type	Shot No.	Charge size, lb.	$\bar{r}$	Travel distance ft.	Travel velocity, ft./sec.	Initial vertical acceleration, ft./sec.^{2}	Initial pulse frequency, c.p.s.	$\frac{D}{\bar{r}}$	$a\rho\bar{r}$, lb./ft.^{2}
Statham acceleration........	9	6.25	1.84	27	6,190	696	167	14.7	4,470
GE acceleration.............				27		696	167	14.7	4,470
Gulton acceleration.........				27		660	139	14.7	4,240
Do..................				151		5.67	78	82.1	36.4
Do..................				292		1.45	64	161	9.31
Differentiated MB velocity..				315		.644	-	171	4.14
Statham acceleration........	1	12.5	2.32	28	5,990	1,410	167	12.1	11,400
Do..................				28		1,190	172	12.1	9,640
GE acceleration.............				28		1,070	167	12.1	8,660
Gulton acceleration.........				78		62.5	86	33.6	506
Differentiated MB velocity..				78		60.5	-	33.6	490
Gulton acceleration.........				152		1/ 39.3	106	65.5	1/ 318
Do..................				300		4.73	109	129	38.3
Differentiated MB velocity..				700		.660	-	302	5.34
Statham acceleration........	6	12.5	2.32	28	5,720	1,550	208	12.1	12,600
Do..................				28		1,530	208	12.1	12,400
Differentiated MB velocity..				78		104	-	33.6	842
Gulton acceleration.........				78		94.0	147	33.6	761
Do..................				152		6.92	111	65.5	56.0
Differentiated MB velocity..				301		1.80	-	130	14.6
Gulton acceleration.........				532		1.84	100	229	14.9
Differentiated MB velocity..				532		1.46	-	229	11.8
Gulton acceleration.........	2	25	2.92	77.4	6,110	75.0	83	26.5	764
Differentiated MB velocity..				77.4		72.0	-	26.5	734
Gulton acceleration.........				151		28.6	100	51.7	291
Differentiated MB velocity..				300		1/ 3.74	-	103	1/ 38.1
Gulton acceleration.........				700		1.35	83	240	13.8
GE acceleration.............	4	25	2.92	25.6	5,900	6,150	357	8.77	62,700
Statham acceleration........				25.6		1/ 5,920	417	8.77	1/ 60,300
Do..................				25.6		4,570	455	8.77	46,600
Gulton acceleration.........				76		88.9	96	26.0	906
Differentiated MB velocity..				76		80.8	-	26.0	823
Gulton acceleration.........				150		19.9	75	51.4	203
Differentiated MB velocity..				300		2.24	-	103	22.8
Gulton acceleration.........				699		1.02	83	239	10.4
Differentiated MB velocity..				699		.795	-	239	8.10
Statham acceleration........	3	37.5	3.35	26.5	6,100	2,110	139	7.90	24,700
GE acceleration.............				26.5		1,560	179	7.90	18,200
Differentiated MB velocity..				77		1/ 129	-	23.0	1/ 1,510
Gulton acceleration.........				151		21.7	63	45.1	254
Do..................				700		1.35	63	209	15.8
Differentiated MB velocity..				1,000		1.21	-	299	14.1
Statham acceleration........	5	37.5	3.35	25	5,850	5,960	278	7.46	69,700
GE acceleration.............				25		5,920	417	7.46	69,200
Differentiated MB velocity..				76		110	-	22.7	1,290
Gulton acceleration.........				76		106	125	22.7	1,240
Do..................				151		16.1	109	45.1	188
Differentiated MB velocity..				300		5.35	-	98.6	62.5
Do..................				549		2.16	-	164	25.3
Gulton acceleration.........				549		1.72	86	164	20.1
GE acceleration.............	7	50	3.68	27.2	5,690	4,310	294	7.39	55,400
Differentiated MB velocity..				79		615	-	21.4	7,900
Gulton acceleration.........				79		1/ 288	185	21.4	1/ 3,700
Do..................				151		63.4	100	41.0	814
Differentiated MB velocity..				312		3.86	-	84.8	49.6
Gulton acceleration.........				420		3.93	76	114	50.5
Statham acceleration........	8	75	4.22	26.3	6,030	1/ 9,600	417	6.23	1/ 141,000
GE acceleration.............				26.3		6,020	294	6.23	88,700
Gulton acceleration.........				77		328	100	18.2	4,830
Do..................				151		26.8	89	35.8	395
Differentiated MB velocity..				310		6.25	-	73.5	92.0
Statham acceleration........	10	100	4.64	25.4	5,730	1/ 17,600	455	5.47	1/ 285,000
GE acceleration.............				25.4		15,100	357	5.47	245,000
Gulton acceleration.........				76		750	217	16.3	12,100
Do..................				150		91.1	250	32.3	1,480
Differentiated MB velocity..				300		8.40	-	64.7	136
Do..................				401		6.38	-	86.4	103

1/ Estimated.

The above analysis suffices to show the agreement between directly measured and derived quantities for measurements taken at the same point. Propagation laws for peak amplitudes are required when measurements are taken at various distances from the shot point. Dimensional analysis was employed to develop scaling laws associated with the propagation equations for peak amplitudes. The basic variables and their dimensions, which affect the transient displacement in the rock resulting from the detonation of an explosive charge in a cavity in the rock, are listed in table 5.

TABLE 5. - Variables and their dimensions

Quantity	Symbol	Dimensions
Displacement....................	u	L
Volume of charge................	V	L^3
Pressure at cavity wall..........	P	$ML^{-1} T^{-2}$
Propagation velocity in rock.....	c	LT^{-1}
Time............................	t	T
Travel distance..................	D	L
Density of rock..................	ρ	ML^{-3}

There are 7 variables and only 3 dimensions (M, L, and T); therefore, according to the π theorem[9] there are 4 independent dimensionless ratios or π functions that can be formed from the 7 variables, and the functional relationship between these ratios can be written as

$$\frac{u(t)}{V^{1/3}} = F\left[\frac{P}{\rho c^2}, \frac{ct}{V^{1/3}}, \frac{D}{V^{1/3}}\right] , \qquad (1)$$

where u(t) is the displacement as a function of time. The same explosive was used in all tests; therefore, the charge weight is proportional to the charge volume. Thus, the cube root of the charge weight in pounds, set numerically equal to a length $\bar{r}$ in feet, is proportional to $V^{1/3}$. The quantity $\bar{r}$ rather than $V^{1/3}$ is the scaling parameter used in this paper. Replacing $V^{1/3}$ with $\bar{r}$, equation (1) becomes

$$\frac{u(t)}{\bar{r}} = F\left[\frac{P}{\rho c^2}, \frac{ct}{\bar{r}}, \frac{D}{\bar{r}}\right] \qquad (2)$$

9/ Bridgman, P. W., Dimensional Analysis: Yale University Press, revised ed., 1931, pp. 36-47.

Differentiating equation (2), the particle velocity, v(t), and acceleration, a(t), as functions of time are

$$\frac{v(t)}{c} = F' \left[\frac{P}{\rho c^2}, \frac{ct}{\bar{r}}, \frac{D}{\bar{r}} \right] , \qquad (3)$$

$$\frac{a\bar{r}(t)}{c^2} = F'' \left[\frac{P}{\rho c^2}, \frac{ct}{\bar{r}}, \frac{D}{\bar{r}} \right] , \qquad (4)$$

where F' and F" are the new functions obtained by differentiating F with respect to time. The quantities on the left of equations (2), (3), and (4) are referred to as scaled displacement, velocity, and acceleration, respectively.

Since Sharpe[10/] has shown by theoretical means that the ratio $P/\rho c^2$ enters the equation as the first power, it is assumed that this factor can be taken from the unknown function and placed in the equations as a simple factor. The ratio $ct/\bar{r}$ in equations (2), (3), and (4) gives the time effect for the transient pulse; when only peak values are considered, this factor is assumed a constant. Thus, the scaling equations reduce to

$$\frac{u}{\bar{r}} = \frac{P}{\rho c^2} F_1 \left(\frac{D}{\bar{r}}\right) \qquad (5)$$

$$\frac{v}{c} = \frac{P}{\rho c^2} F_2 \left(\frac{D}{\bar{r}}\right) \qquad (6)$$

$$\frac{a\bar{r}}{c^2} = \frac{P}{\rho c^2} F_3 \left(\frac{D}{\bar{r}}\right) . \qquad (7)$$

The scaled amplitudes of displacement, velocity, and acceleration are constant at given scaled distances only when $P/\rho c^2$ is constant. The methods of loading and stemming the charges were the same in all tests; therefore, P is assumed constant for a given rock type. However, there were observed differences in c; thus, equations (5), (6), and (7) are rewritten as follows:

10/ Sharpe, J A., The Production of Elastic Waves by Explosive Pressures; I Theory and Empirical Field Observations: Geophysics, vol. 7, No. 3, 1942, pp.144-154.

$$\frac{u\,\rho c^2}{\bar{r}} = P\,F_1\left(\frac{D}{\bar{r}}\right) \qquad (8)$$

$$v\,\rho c = P\,F_2\left(\frac{D}{\bar{r}}\right) \qquad (9)$$

$$a\bar{r}\,\rho = P\,F_3\left(\frac{D}{\bar{r}}\right) \quad . \qquad (10)$$

The left hand quantities are referred to as the reduced displacement, velocity, and acceleration since all three have been reduced to pressure units. These are the quantities which are plotted as functions of scaled distances to determine the form of the unknown functions F_1, F_2, and F_3.

Log-log plots of reduced displacement, velocity, and acceleration versus the parameter $(D/\bar{r})$ are shown in figures 8, 9, and 10, respectively. These figures include data obtained both from direct measurement, and graphical derivation, and different symbols are used to distinguish the measurements obrained from the displacement meter, the velocity gages, and the accelerometers. The data for all charge sizes tend to group about a smooth curve indicating that the scaling laws are correct. The derived data fall within the error limits of the directly measured data; therefore, both sets of data will give approximately the same propagation law.

The propagation equations for displacement, velocity, and acceleration, determined by the least square methods, are of the form:

$$\frac{u\,\rho c^2}{\bar{r}} = K_1\left(\frac{D}{\bar{r}}\right)^{n_1} \qquad (11)$$

$$v\,\rho c = K_2\left(\frac{D}{\bar{r}}\right)^{n_2} \qquad (12)$$

$$a\,\rho\,\bar{r} = K_3\left(\frac{D}{\bar{r}}\right)^{n_3} \quad . \qquad (13)$$

The values of the constants K and n and their standard deviations are given in table 6. Equations (11), (12), and (13) are shown plotted in figures 8, 9, and 10 as heavy solid lines. The limits of plus or minus one standard deviation, which include 66 percent of the data, are also shown in these figures. The data spread in any one plot is substantially unaffected by either omission or inclusion of the derived points.

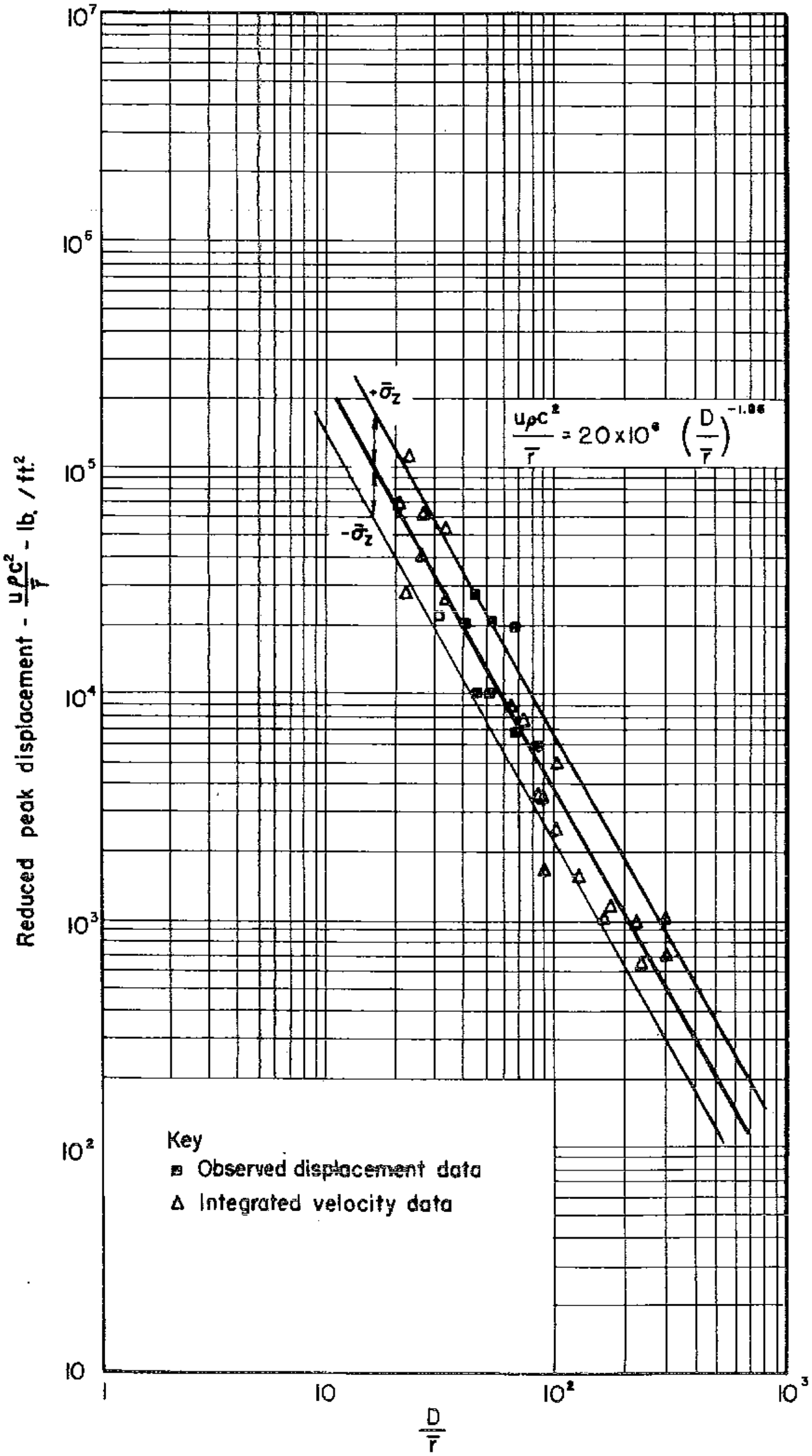


Figure 8. - Reduced peak displacement vs. $\frac{D}{r}$.

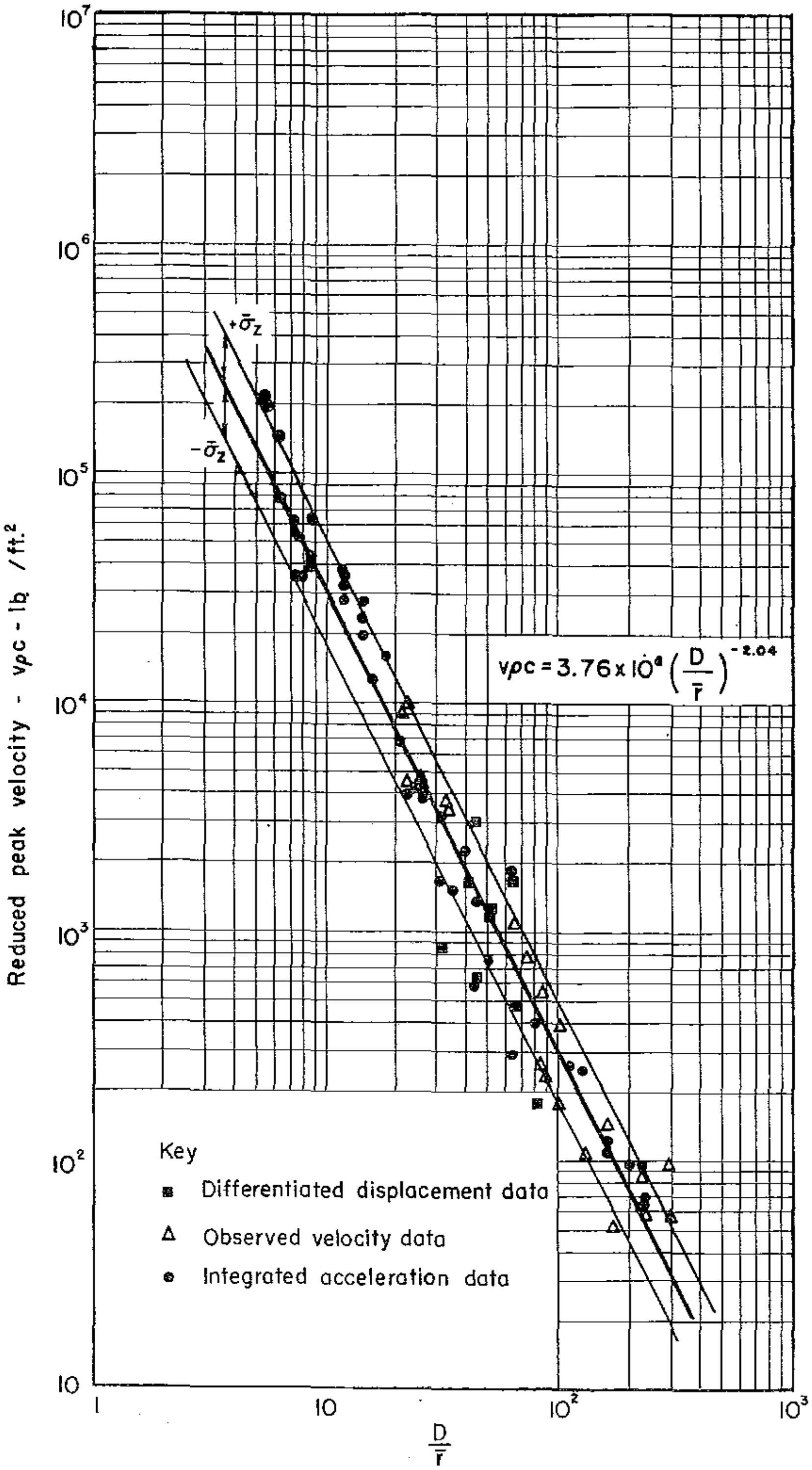


Figure 9. - Reduced peak velocity vs. $\frac{D}{r}$.

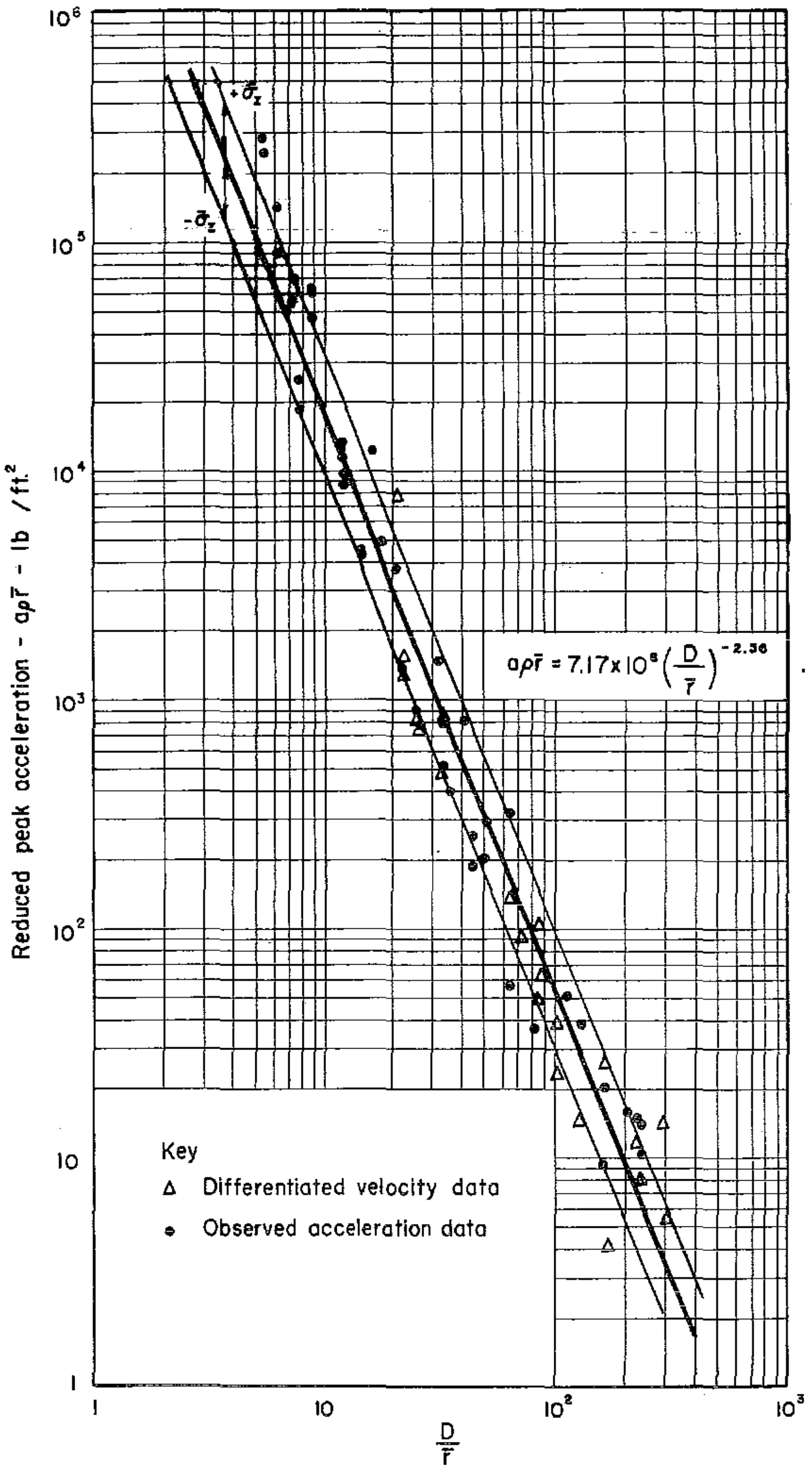


Figure 10. - Reduced peak acceleration vs. $\frac{D}{\bar{r}}$.

TABLE 6. - Propagation law constants and their standard deviations

	Slope		Intercept		Standard error, estimate $\bar{\sigma}_z$	
	n	$\bar{\sigma}_n$	K	$\bar{\sigma}_k$		Percent
Reduced displacement	-1.86	±0.13	20×10^6	$+14.4 \times 10^6$ $- 8.3 \times 10^6$	0.23	+70 -41
Reduced velocity	-2.04	± .05	3.76×10^6	$+ .78 \times 10^6$ $- .64 \times 10^6$	.205	+61 -37
Reduced acceleration	-2.56	± .07	7.17×10^6	$+2.01 \times 10^6$ -1.58×10^6	.274	+88 -47

CONCLUSIONS

1. The General Electric, Gulton, and Statham accelerometers gave good inter and intra group reproducibility. The MB velocity gages showed good intra group reproducibility.

2. The agreement between the derived (differentiated and/or integrated) and the directly observed records established that the gage records are true representation of the particle motion of the rock.

3. The experimental data agree with scaling laws developed by dimensional analysis.

4. The propagation laws for peak amplitudes of displacement, velocity, and acceleration are independent of gage type.